

Moodle Exercise Set 8

Calculus and Linear Algebra II

Spring 2020

1. Solve the differential equation $y' = (1 + y^2)(4x^3 + 2x)$.
2. Solve the differential equation $y' + \frac{y}{x} = \cos(x^2)$.
3. Find the solution of the initial value problem $x^2 y' + xy = 1$ for $x > 0$ and $y(1) = 2$.
4. Solve the differential equation $y'' = -4y$.
5. Solve the second order differential equation $xy'' + 2y' = 12x^2$.
6. A population is modelled by the differential equation $P' = 1.2P(1 - \frac{P}{4200})$. For what initial values of P do we get equilibrium solutions?
7. Consider an electrical circuit that contains a battery, a resistor with resistance R ohms (Ω), an inductor of L henries (H), a battery of voltage E volts (V), and a switch. At time t , the battery produces a current of $I(t)$ amperes (A). The relationship between these quantities in the circuit is given by the differential equation

$$L \frac{dI}{dt} + RI = E.$$

Suppose $R = 12\Omega$, $L = 4\text{H}$, and $E = 60\text{V}$. Identify any equilibrium solutions for the current. Draw a direction field and determine which of the equilibrium solutions are stable. What can you say about the limiting value (i.e., $t \rightarrow \infty$) of the current?

8. A 4×4 invertible matrix A has determinant $\det(A) = \frac{1}{2}$. Find $\det(2A)$, $\det(-A)$, $\det(A^2)$, and $\det(A^{-1})$.
9. Find the determinant of the matrix

$$Q_\theta = \begin{bmatrix} 1 - 2\cos^2(\theta) & -2\cos(\theta)\sin(\theta) \\ -2\cos(\theta)\sin(\theta) & 1 - 2\sin^2(\theta) \end{bmatrix}.$$

10. A rotation about the y -axis by an angle θ in \mathbb{R}^3 is described by the matrix

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}.$$

What is $\det(R_y(\theta))$?

11. Find the eigenvalues (with multiplicities) of the matrix

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

12. Let A be an invertible 2×2 matrix with eigenvalue λ . Furthermore let μ be any real number. Find the eigenvalues of $B = (I + \mu A^{-1})$ where I is the identity matrix.
13. Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. What are the eigenvalues and eigenvectors of A^2 ?

14. Find the eigenvalues and eigenvectors of the matrix

$$S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

15. Consider the matrix $A = \frac{1}{10} \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}$. Diagonalize the matrix to find $\lim_{k \rightarrow \infty} A^k$.