1. Solve the differential equation \( y' = (1 + y^2)(4x^3 + 2x) \).

2. Solve the differential equation \( y' + \frac{y}{x} = \cos(x^2) \).

3. Find the solution of the initial value problem \( x^2y' + xy = 1 \) for \( x > 0 \) and \( y(1) = 2 \).

4. Solve the differential equation \( y'' = -4y \).

5. Solve the second order differential equation \( xy'' + 2y' = 12x^2 \).

6. A population is modelled by the differential equation \( P' = \frac{1}{2}P(1 - \frac{P}{4200}) \). For what initial values of \( P \) do we get equilibrium solutions?

7. Consider an electrical circuit that contains a battery, a resistor with resistance \( R \) ohms (\( \Omega \)), an inductor of \( L \) henries (\( H \)), a battery of voltage \( E \) volts (\( V \)), and a switch. At time \( t \), the battery produces a current of \( I(t) \) amperes (\( A \)). The relationship between these quantities in the circuit is given by the differential equation

\[
L\frac{dI}{dt} + RI = E.
\]

Suppose \( R = 12\Omega, \ L = 4H, \ \text{and} \ E = 60V \). Identify any equilibrium solutions for the current. Draw a direction field and determine which of the equilibrium solutions are stable. What can you say about the limiting value (i.e., \( t \to \infty \)) of the current?

8. A \( 4 \times 4 \) invertible matrix \( A \) has determinant \( \det(A) = \frac{1}{2} \). Find \( \det(2A), \det(-A), \det(A^2), \) and \( \det(A^{-1}) \).

9. Find the determinant of the matrix

\[
Q_\theta = \begin{bmatrix}
1 - 2\cos^2(\theta) & -2\cos(\theta)\sin(\theta)
\end{bmatrix}
\]

10. A rotation about the \( y \)-axis by an angle \( \theta \) in \( \mathbb{R}^3 \) is described by the matrix

\[
R_y(\theta) = \\
\begin{bmatrix}
\cos(\theta) & 0 & \sin(\theta) \\
0 & 1 & 0 \\
-\sin(\theta) & 0 & \cos(\theta)
\end{bmatrix}
\]

What is \( \det(R_y(\theta)) \)?

11. Find the eigenvalues (with multiplicities) of the matrix

\[
M = \begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}
\]

12. Let \( A \) be an invertible \( 2 \times 2 \) matrix with eigenvalue \( \lambda \). Furthermore let \( \mu \) be any real number. Find the eigenvalues of \( B = (I + \mu A^{-1}) \) where \( I \) is the identity matrix.

13. Let \( A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \). What are the eigenvalues and eigenvectors of \( A^2 \)?
14. Find the eigenvalues and eigenvectors of the matrix

\[ S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \]

15. Consider the matrix \( A = \frac{1}{10} \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix} \). Diagonalize the matrix to find \( \lim_{k \to \infty} A^k \).