Foundations of Mathematical Physics

Organization:

- class: Wed/Fri, 9:45-11:00 (WH 8)
- website: lecture notes, homework, news
- grade: 100% final exam
  - ↳ bonus: 10% bonus from HW sheets (average HW score divided by 10) (not counting the 2 worst HW sheets)
  - ↳ note: bonus cannot change "fail" to "pass" grade
- TA: Martin Irmgu
- books:
  - lecture notes by Stefan Teufel (U Tübingen) } ~ first half of class
  - further references on website } for ~ second half of class

Topics:

Generally: Mathematics of (non-relativistic) quantum mechanics (QM)

Fields we touch upon:

↳ Math:
- Analysis
- Functional Analysis (Linear Algebra)
- PDEs

↳ Physics:
- Schrödinger eq.
- many-body QM
- Bose-Einstein condensation (BEC)
- cold atoms, condensed matter physics

Structure of this class:

- short introduction to quantum mechanics
- Fourier transform, distributions, free Schrödinger equation
- Hilbert space, self-adjoint operators, unitary groups, interacting Schrödinger eq.
  (stop at bounded operators; briefly discuss results for unbounded operators)
- non-linear Schrödinger equation, second quantization (Fock space), BEC, Bogolivbov theory

1. Introduction

1.1 Motivation

classical physics: • Newton's eq.: $F = m \cdot a$

↳ initial position and momentum determine trajectories of point particles

• with special (general) relativity: locality

↳ "nothing moves faster than the speed of light c"

problems at beginning of 20th century: • blackbody radiation

• photoelectric effect

• stability of atoms / matter

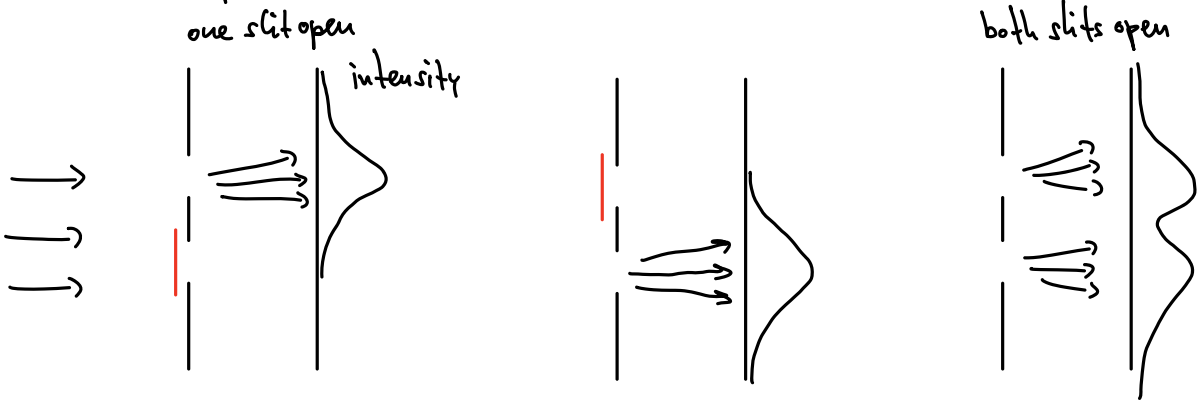
solution to these problems led to Quantum Mechanics (QM)

↳ physics: Planck, Einstein, Schrödinger, Bohr, Heisenberg, Dirac, Born, de Broglie

↳ math: Hilbert, von Neumann

Example of "weird" quantum behavior: double slit experiment

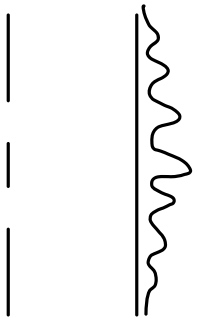
- classical particles:



- electromagnetic waves (light), water waves:

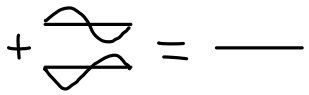
single slit similar

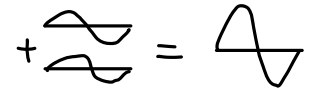
both slits open



interference pattern

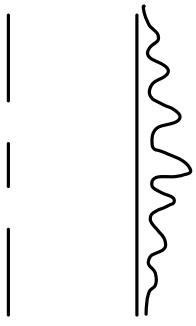
explanation:





- electrons:

both slits open

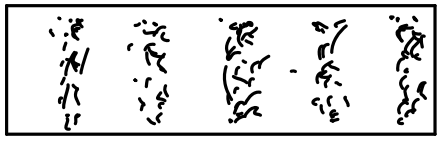


interference, big surprise!

maybe a collective wave?

single separated electrons: (both slits open)

side view



single localized detection events => electrons are particles

interference pattern => electrons are waves and go through both slits at the same time

=> seems like a paradox

Resolution of this paradox still hotly debated

Not debated: probabilistic description: some wave describes probability distribution of particle positions