Jacobs University
Fall 2021

September 13, 2021

Operations Research

Homework 2

Due on September 20, 2021

Note: Your homework must be submitted via moodle (see the link on the class website) on the due day BEFORE THE TUTORIAL.

Problem 1 [10 points]
Find all solutions for the underdetermined linear system $A\boldsymbol{x} = \boldsymbol{b}$, where

$$A = \begin{pmatrix} 2 & 2 & -1 & -4 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 1 & 3 & 5 \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{b} = \begin{pmatrix} -5 \\ 2 \\ -1 \\ 8 \end{pmatrix} .$$

Problem 2 [5 points]
Reconsider Problem 1 above: State at least two different basic solutions. Make sure that at least one of these is a basic *feasible* solution, i.e., a solution where all components are non-negative.

Problem 3 [5 points]
Reconsider Problem 1 from Homework Sheet 1: *Minimize*

$$z = 8\,x_1 + 12\,x_2$$

subject to

$$\begin{gathered} 5\,x_1 + 2\,x_2 \geq 20\,, \\ 4\,x_1 + 3\,x_2 \geq 24\,, \\ x_2 \geq 2\,, \\ x_1, x_2 \geq 0\,. \end{gathered}$$

Introduce slack variables to write this linear programming problem in the standard form: *Minimize*

$$z = \boldsymbol{c}^T \boldsymbol{x}$$

subject to

$$\begin{gathered} A\boldsymbol{x} = \boldsymbol{b} \\ \boldsymbol{x} \geq 0 \end{gathered}$$

where the coefficients $\boldsymbol{b}, \boldsymbol{c}$, and the decision variables $\boldsymbol{x}$ are written as column vectors, and A is a matrix of matching dimension.