# Operations Research 

## Homework 7

Due on October 25, 2021

Note: Your homework must be submitted via moodle (see the link on the class website) on the due day BEFORE THE TUTORIAL.

## Problem 1 [6 points]

A construction company needs a heavy excavator for a three-year project. Since maintenance is expensive, it may be cheaper to replace the machine in the interim. The overall net cost (in $1000 €$ ) of purchasing the excavator at the beginning of year $i$ and trading it at the end of year $j$ is as follows.

|  | Trade in end of year |  |  |
| :---: | :---: | :---: | :---: |
| Purchase start of year | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| $\mathbf{1}$ | 25 | 60 | 95 |
| $\mathbf{2}$ | - | 29 | 65 |
| $\mathbf{3}$ | - | - | 37 |

The task is to find the most economical schedule of purchasing and trading in the excavator(s).
(a) Draw a graphical representation and show that this is a shortest path problem.
(b) Solve the problem. This is most easily done by hand in this case.

## Problem 2 [8 points]

(a) Write a Pyomo program to solve the maximum flow problem for the Seervada Park example from Hillier and Lieberman, with the network data reproduced here. The arcs are directed and have a flow capacity as indicated.
(b) Ask the solver to return dual variables as well. Print out the dual variables that correspond to the capacity constraints. (Note: This can be used to identify the minimum cut of the network; see HL (9th edition) Chapter 9.5 "Finding an Augmenting Path".)


## Problem 3 [6 points]

(HL, Problem 9.4-3.) The Premiere Bank soon will be hooking up computer terminals at each of its branch offices to the computer at its main office using special phone lines with telecommunications devices. The phone line from a branch office need not be connected directly to the main office. It can be connected indirectly by being connected to another branch office that is connected (directly or indirectly) to the main office. The only requirement is that every branch office be connected by some route to the main office. The charge for the special phone lines is $100 \$$ times the number of miles involved, where the distance (in miles) between every pair of offices is as follows:

|  | Distance between Pairs of Offices |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Main | B.1 | B.2 | B.3 | B.4 | B.5 |  |
| Main Office | - | 190 | 70 | 115 | 270 | 160 |  |
| Branch 1 | 190 | - | 100 | 110 | 215 | 50 |  |
| Branch 2 | 70 | 100 | - | 140 | 120 | 220 |  |
| Branch 3 | 115 | 110 | 140 | - | 175 | 80 |  |
| Branch 4 | 270 | 215 | 120 | 175 | - | 310 |  |
| Branch 5 | 160 | 50 | 220 | 80 | 310 | - |  |

Management wishes to determine which pairs of offices should be directly connected by special phone lines in order to connect every branch office (directly or indirectly) to the main office at a minimum total cost.
(a) Describe how this problem fits the network description of the minimum spanning tree problem.
(b) Use the algorithm discussed in class to solve the problem.

