

Operations Research

Homework 11

Due on November 29, 2021

Note: Your homework must be submitted via moodle (see the link on the class website) on the due day BEFORE THE TUTORIAL.

Problem 1 [7 points]

(HL, Exercise 18.7-8.) Suppose that the demand D for a spare airplane part has an exponential distribution with mean 50, that is,

$$\varphi_D(x) = \begin{cases} \frac{1}{50}e^{-x/50} & \text{for } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

This airplane will be obsolete in 1 year, so all production of the spare part is to take place at present. The production costs now are \$1 000 per item—that is, $c = 1\,000$ —but they become \$10 000 per item if they must be supplied at later dates—that is, $p = 10\,000$. The holding costs, charged on the excess after the end of the period, are \$300 per item.

- Determine the optimal number of spare parts to produce.
- Suppose that the manufacturer has 23 parts already in inventory (from a similar, but now obsolete airplane). Determine the optimal inventory policy.
- Suppose that p cannot be determined now, but the manufacturer wishes to order a quantity so that the probability of a shortage equals 0.1. How many units should be ordered?
- If the manufacturer were following an optimal policy that resulted in ordering the quantity found in part (c), what is the implied value of p ?

Problem 2 [7 points]

- Use the graphical method to maximize

$$Z = x_1 + 2x_2$$

subject to

$$\begin{aligned} x_1^2 + x_2^2 &\leq 1, \\ x_1, x_2 &\geq 0. \end{aligned}$$

- (b) Write a Pyomo program to confirm your answer. (Use the `ipop` solver instead of `glpk`.)

Problem 3 [6 points]

(*HL, Exercise 12.2-3.*) Consider the variation of the Wyndor Glass Co. example where the second and third functional constraints of the original problem have been replaced by $9x_1^2 + 5x_2^2 \leq 216$. Demonstrate that $(x_1, x_2) = (2, 6)$ with $Z = 36$ is indeed optimal by showing that the objective function line $36 = 3x_1 + 5x_2$ is tangent to this constraint boundary at $(2, 6)$. (Hint: Express x_2 in terms of x_1 on this boundary, and then differentiate this expression with respect to x_1 to find the slope of the boundary.)