Operations Research, Jacobs University, Fall 2021 — Session 1

Prof. Sören Petrat — Sep. 2, 2021

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Class organization:

- website (all infos, lecture notes), MS Teams (class recordings, some online sessions), moodle (HW submission)

- class: • Tue, Thu: 15:45-17:00, in-person
  • recordings / live streams on MS Teams

- homework:
  ↳ ~ weekly assignments (start next week)
  ↳ available on moodle and website
  ↳ submission only via moodle (week after)
  ↳ solution discussed in tutorial

- grade: • final exam only
  • bonus: up to 5% from HW, up to 5% from midterm
    ↳ HW: average of all but 2 worst HW sheets, score above 50%, divided by 10.
    ↳ midterm: score above 50 %, divided by 10.
    Important: bonus cannot change fail grade to pass grade!

- TAs: Ghita Choqi, Mohcine Khatori
  ↳ weekly tutorials, question sessions
  ↳ grading

– textbook: Hillier, Lieberman – Introduction to Operations Research

+ see website

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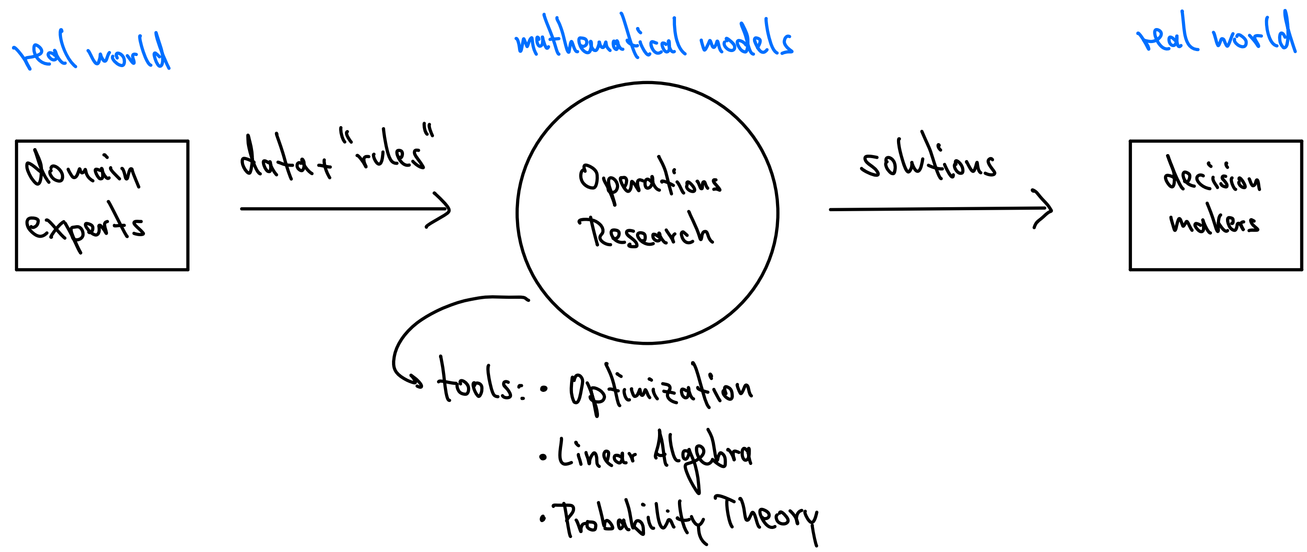
1. Introduction

Operations Research (OR):

↳ scientific approach to management/planing problems for organizations

↳ input: from many different departments; output: optimal solution(s)

↳ has led to immense savings



Key steps in OR problems:

1. Definition of the problem
2. Gather relevant data
3. Formulate a mathematical model
4. Solve the model (usually computer-based)
5. Test the model, sensitivity analysis
6. Recommendation and/or implementation

Typical models / topics of this class:

- linear programs (better: linear optimization problems) ⇙ $\frac{1}{2}$ of this class
  - ↳ here: examples + theory ; computer implementation with pyomo library in Python
  - ↳ includes network optimization
- Nonlinear programs
- Dynamic programs (can be linear or nonlinear)
- Decision theory (involves probability)
- Inventory theory
- (Queueing theory)

Today, we start with a prototypical example (see also Hillier/Lieberman Ch. 3):

Wyndor Glass Co.

1. Problem setup:

↳ 3 plants:
- Plant 1: aluminium frames
- Plant 2: wood frames
- Plant 3: glass + final assembly

↳ 2 (new) products:
- Product 1: glass door with aluminium frame
- Product 2: wood-framed window

↳ Assume all that can be produced can be sold (marketing).

↳ Task: How many units of products 1 and 2 should be made to maximize profit, subject to the available capacities?

2. Data:

	required production time per batch (in hours)		available production time (in hours per week)
	Product 1	Product 2	
Plant 1	1	0	4
Plant 2	0	2	12
Plant 3	3	2	18
profit per batch	3000 $	5000 $	

3. Mathematical model:

↳ Decision variables: • x_1 = # of batches of product 1

• x_2 = # of batches of product 2

↳ Objective function (here: profit): $Z = 3x_1 + 5x_2$ (in k $), to be maximized

↳ Constraints: • $x_1, x_2 \geq 0$ (0)

• $x_1 \leq 4$ (1)

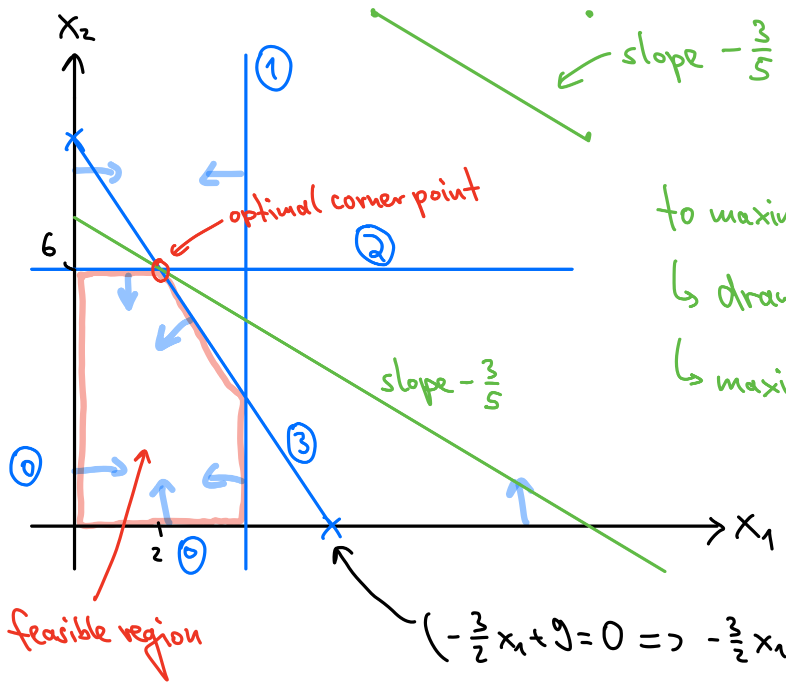
• $2x_2 \leq 12$ (2) $(x_2 \leq 6)$

• $3x_1 + 2x_2 \leq 18$ (3)

↳ solve for x_2: $2x_2 \leq -3x_1 + 18$

$\Rightarrow x_2 \leq -\frac{3}{2}x_1 + 9$

4. Graphical solution:



to maximize: $Z = 3x_1 + 5x_2$

↳ draw $x_2 = \underbrace{\tfrac{1}{5}}_{}Z - \frac{3}{5}x_1 \Rightarrow$ line with slope $-\frac{3}{5}$

↳ maximize intercept with x_2 axis

$\left(-\frac{3}{2}x_1 + 9 = 0 \Rightarrow -\frac{3}{2}x_1 = -9 \Rightarrow x_1 = 9 \cdot \frac{2}{3} = 6\right)$

optimal corner point: • read off: $x_1 = 2,\ x_2 = 6$

• or: ②: $2x_2 = 12 \Rightarrow x_2 = 6$

③: $3x_1 + 2\underbrace{x_2}_{=6} = 18 \Rightarrow 3x_1 = 6 \Rightarrow x_1 = 2$

6. Recommendation:

↳ produce 2 batches of product 1, and 6 batches of product 2

↳ then profit will be maximal, namely

$$Z = 3x_1 + 5x_2 = 3 \cdot 2 + 5 \cdot 6 = 36 \quad (k\$)$$