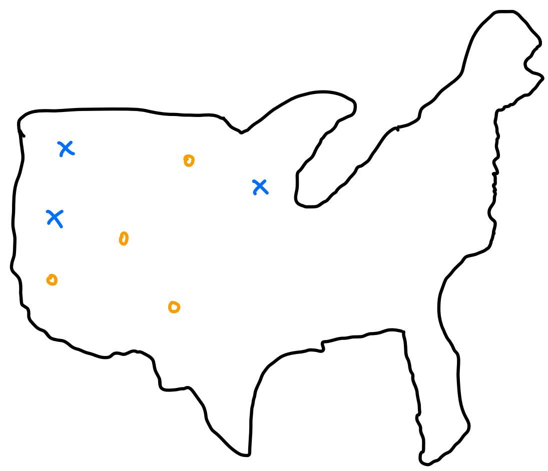
2.5 Transportation Problems

Example (Hillier, Lieberman Chapter 8): P&T company
↳ canned peas are prepared at canneries (x) in 3 cities across the US
↳ then shipped to 4 warehouses (o) across the US

Goal: minimize shipping cost but ensure supply



Data:

		Shipping cost per truckload				Output
		Warehouses				
		1	2	3	4	
Cannery	1	c_{11}	c_{12}	c_{13}	c_{14}	S_1
	2	c_{21}	c_{22}	c_{23}	c_{24}	S_2
	3	c_{31}	c_{32}	c_{33}	c_{34}	S_3
Allocation		d_1	d_2	d_3	d_4	

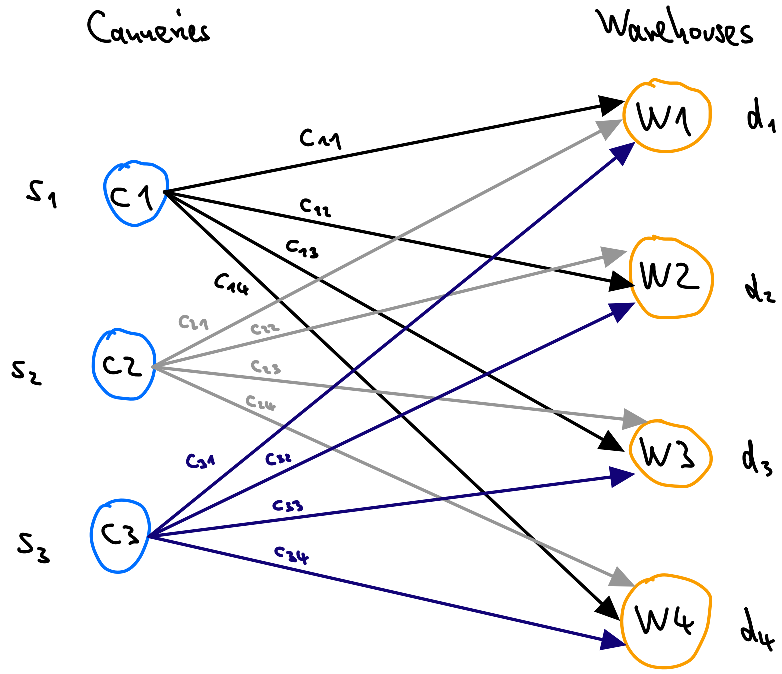
c_{ij}

S_i, $i = 1, \dots, m$ ($m = 3$ here)

d_j, $j = 1, \dots, n$ ($n = 4$ here)

decision variables: x_{ij} = number of truckloads shipped from cannery i to warehouse j

Network view of the transportation problem:



This leads to the following LP transportation problem:

- $\boxed{\text{Minimize}}$ transportation cost $Z = c_{11}x_{11} + c_{12}x_{12} + \ldots + c_{mn}x_{mn} = \boxed{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}}$
- subject to: $x_{i1} + x_{i2} + x_{i3} = s_i$ (everything is shipped away from cannery i)

  i.e., $\underbrace{\boxed{\sum_{j=1}^{n} x_{ij} = s_i}}$ for all $i = 1, \ldots, m$ (all canneries)

  might want to relax this to $\sum_{j=1}^{n} x_{ij} \le s_i$ (at most as much as we have is shipped away)
  ↳ see next class

  and $x_{1j} + x_{2j} + x_{3j} + x_{4j} = d_j$ (warehouse j receives the necessary supply)

  i.e., $\underbrace{\boxed{\sum_{i=1}^{m} x_{ij} = d_j}}$ for all $j = 1, \ldots, n$ (all warehouses)

  might want to relax this to $\sum_{i=1}^{n} x_{ij} \ge d_j$ (warehouses receive at least the necessary supply)
  ↳ see next class

  and $\boxed{x_{ij} \ge 0.}$