

## Another example: Project Management

### Problem type 1:

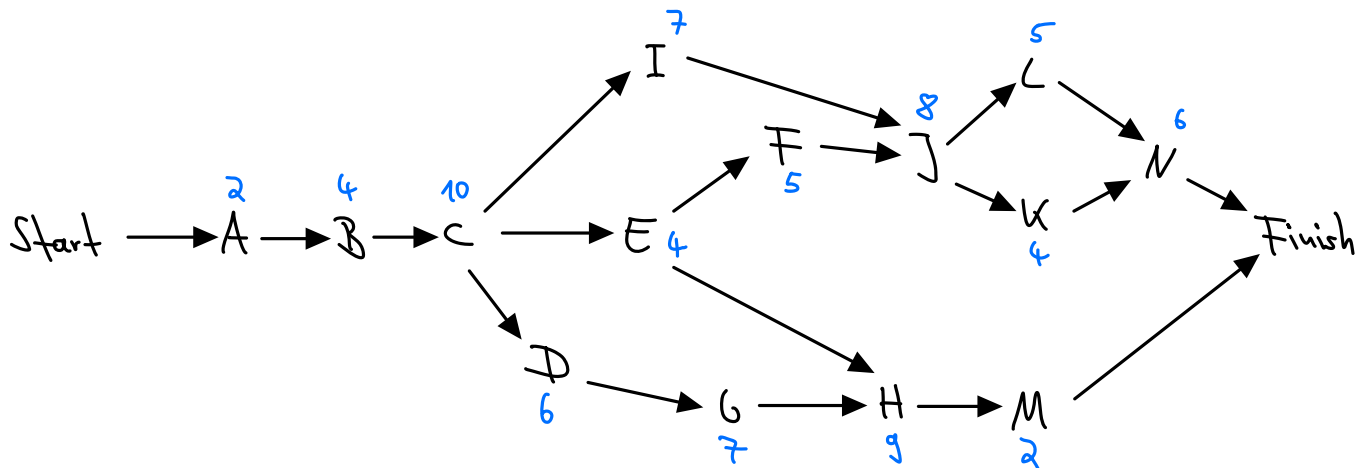
- Given: set of activities taking time  $T_i$  to complete, and their dependencies (e.g., building construction)
- Goal: find minimal time to completion, and the corresponding order of activities (= critical path through network)
- Set up:
  - decision variables  $t_i$  = starting time of activity  $i$
  - minimize  $t_{\text{finish}}$
  - constraints:  $t_j \geq t_i + T_i$  if  $j$  depends on  $i$
  - $t_{\text{start}} = 0$
  - $t_i \geq 0$

### Problem type 2:

- Suppose a completion time is prescribed, but it is shorter than the critical path from above. Assume we can reduce the times of certain activities at a cost (this is called "crashing" an activity).
- Introduce  $x_i$  = units of time saved on activity  $i$  (decision variables)
  - $T_i$  = regular time for completion
  - $R_i$  = maximal time that can be saved
  - $c_i$  = cost of saving one unit of time

- LP problem: minimize cost  $\sum_i c_i x_i$   
subject to  $x_i \leq R_i$  for all activities  $i$   
 $t_j \geq t_i + (T_i - x_i)$   
 $t_i \geq 0, x_i \geq 0$

Example (Hillier, Lieberman Chapter 9.8 (9th edition)): Reliable Construction Company



Critical path = longest path = A-B-C-E-F-J-L-N = 44 weeks  
 so all activities can be finished

Suppose project needs to be completed in 40 weeks, i.e., we need to crash some activities → see promo code discussion.

Some possible exam topics/questions:

- Formulate a given "text problem" as LP
- Solve LP problem graphically (also: shape of feasible region, number of solutions)
- Write LP problem in standard form
- Gaussian elimination and basic solutions
- Use simplex method to solve LP problem (what if feasible region is unbounded?)
- Shadow prices and their meaning
- Dual LP problems, weak and strong duality
- Transportation problems and their LP formulation
- Integer solution property, dummy variables
- Solve shortest path, minimum spanning tree, maximum flow problems
- Minimum cost flow problem and LP
- Pyomo: explain code; explain output; extract LP problem in mathematical notation from code; what happens if something is changed in the code

Good practice midterm: Fall 2017 (see website)