

3. Further Optimization Techniques

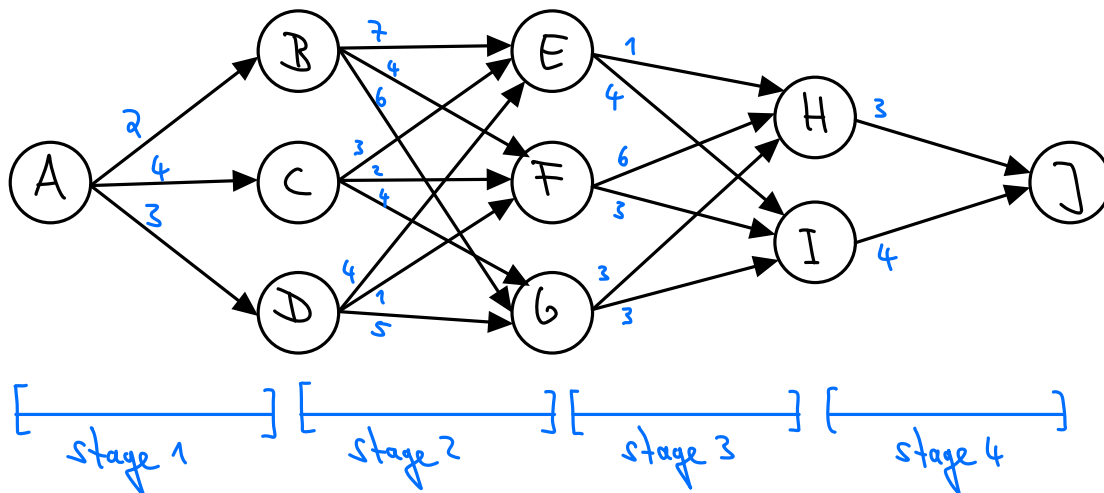
3.1 Dynamic Programming

Setting of Dynamic Programming:

- problem can be divided into several "stages"
- a "policy decision" (= transition from one "state" to another) needs to be made at each stage
- goal: find optimal sequence of decisions (= "optimal policy") ← usually a min. or max. problem
- decision variables: x_i = state to transition to in stage i (from some state at stage $i-1$).

Example: Stagecoach problem (Hillier, Lieberman: Chapter 10.1)

Need to travel from A to J; travel/insurance costs are associated to different route segments:



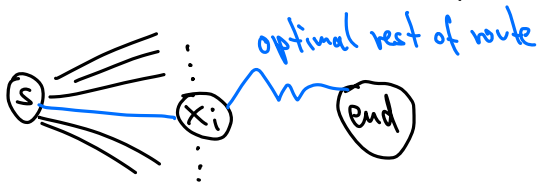
Note: This is a special type of shortest path problem: one with different stages.

Route: $A \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 = J$

Solution procedure: Start from the end and go backwards through the stages.

We introduce:

- $f_i(s, x_i) =$ cost of optimal travel route starting at s at stage $i-1$, passing through x_i at stage i , going to the end



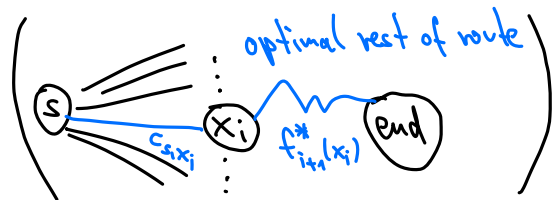
- $f_i^*(s) = \min_{x_i} f_i(s, x_i) =$ cost of optimal travel route starting at s

(let x_i^* denote the minimum (not necessarily unique) = optimal choice at stage i)

• $\Rightarrow f_i(s, x_i) = C_{s, x_i} + f_{i+1}^*(x_i)$

cost of travel from s to x_i in the network above (given parameters)

optimal future cost to travel from x_i to the end



Solution:

stage $i = 4$:

s	f_4^*	x_4^*
H	3	J
I	4	J

$i = 3$:

s	$f_3(s, x_3) = C_{s, x_3} + f_4^*(x_3)$		$f_3^*(s)$	x_3^*
	$x_3 = H$	$x_3 = I$		
E	$1 + 3 = 4$ <small>$C_{E,H}$ $f_4^*(H)$</small>	$4 + 4 = 8$	4	H
F	$6 + 3 = 9$	$3 + 4 = 7$	7	I
G	$3 + 3 = 6$	$3 + 4 = 7$	6	H

$i = 2$

s	$f_2(s, x_2) = C_s x_2 + f_3^*(x_2)$			$f_2^*(s)$	x_2^*
	$x_2 = E$	$x_2 = F$	$x_2 = G$		
B	$7 + 4 = 11$	$4 + 7 = 11$	$6 + 6 = 12$	11	E or F
C	$3 + 4 = 7$	$2 + 7 = 9$	$4 + 6 = 10$	7	E
D	$4 + 4 = 8$	$1 + 7 = 8$	$5 + 6 = 11$	8	E or F

$i = 1$

s	$f_1(s, x_1) = C_s x_1 + f_2^*(x_1)$			$f_1^*(s)$	x_1^*
	$x_1 = B$	$x_1 = C$	$x_1 = D$		
A	$2 + 11 = 13$	$4 + 7 = 11$	$3 + 8 = 11$	11	C or D

\Rightarrow Minimal cost is 11, optimal paths are

A-C-E-H-J or A-D-E-H-J or A-D-F-I-J

Example: Medical Teams

# of medical teams	Countries		
	1	2	3
0	0	0	0
1	45	20	50
2	70	45	70
3	90	75	80
4	105	110	100
5	120	150	130

\leftarrow # of additional person-years of life saved (in thousands)

Goal: distribute 5 medical teams to the three countries s.t. maximum person-years saved.

Solution: next time