Calculus and Elements of dinear Algebra I Session 9
Prof. Soren Petrot, Dr. Stephan Juricle (based on lacture
notes by Marcel Oliver)
Jacobs University, Fall 2022.
2. Derivatives
2.1 Introduction to derivatives and their properties
Topic 2.1.B: Differentiation rules
Addition rule: If f', g' exist, then
$$(f*g)'(x) = f'(x)*g'(x)$$

Product rule: If f', g' exist, then $(f*g)'(x) = f'(x)*g'(x)$
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Product rule: If f', g' exist, then $(f*g)'(x) = f(x)*g'(x)$
Product rule: $(f*g)'(x) = lim_{h=0} \frac{(f*g)(x)+f(x)*g(x)}{h}$
 $= lim_{h=0} \frac{f(x+h)-g(x+h)-f(x+h)g(x)}{h} + g(x) lim_{h=0} \frac{f(x+h)-f(x)}{h}$

 $\lim_{h \to 0} f(x+h) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$ =f(x) = g'(x)be cause f is continuous

 $= f(x) \cdot c'(x) + f'(x) \cdot q(x)$ Note: If derivative of f exists at $x \in (a, b)$, then f is continuous at x. differentiability implies continuity " (but not the other way around) $\underline{Example}$: |X| at x = 0 is continuous but not diff 'able CEIR is a constant: C'=0 Diff. of constant: Multiplication with constant: $(C \cdot f)'(x) = C \cdot f'(x)$

Quotient rule:

$$\begin{pmatrix} f \\ g \end{pmatrix}'(x) = \frac{f'(x) \cdot g(x) - g'(x) f(x)}{(g(x))^2}$$

$$\begin{pmatrix} if \\ g(x) \neq 0 \end{pmatrix}$$

Preef questiont reale:
$$g(x) \cdot \frac{f(x)}{g(x)} = f(x) \quad (if g(x) \neq 0)$$

 $\Rightarrow (g \cdot \frac{g}{g})^{i}(x) = f^{i}(x)$
Product $g^{i}(x) \cdot (\frac{f}{g})(x) + g(x) \cdot (\frac{g}{g})^{i}(x) = f^{i}(x)$
 $preduct g^{i}(x) \cdot (\frac{f}{g})(x) + g(x) \cdot (\frac{g}{g})^{i}(x) = f^{i}(x)$
 $preduct g^{i}(x) = \frac{f}{g(x)} (f^{i}(x) - g^{i}(x)(\frac{f}{g})(x)) = \frac{g(x)f^{i}(x) - g^{i}(x)f^{k}}{(g(x))^{2}}$
Chain reale: $(f(g(x)))^{i} = f^{i}(g(x)) \cdot g^{i}(x)$
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 $preof chain reale: in live lecture
Preof chain reale: $n \in \mathbb{N}$, $(x^{n})^{i} = n \times n^{n-4}$
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 $n = \Lambda : x^{n} = x$, $x^{i} = \lim_{h \to 0} \frac{x + h - x}{h} - \lim_{h \to 0} \frac{h}{h} = 1$
 $= \Lambda \cdot x^{n} = \Lambda \cdot x^{n} + x \cdot (x^{n})^{i}$
 $= x^{n} + x \cdot n \times x^{n-4}$
 $= x^{n} + n \cdot x^{n} = (n+4)x^{n}$$

$$\begin{cases} \Rightarrow \text{ We can extend the power rule to rational exponents. Proof in live lecture.} \\ (x^{\frac{1}{2}})' = \frac{p}{q} \times \frac{p}{1}, \text{ for } p, q \in \mathbb{Z} \\ \\ \text{It can be further extended to arbitrary } r \in \mathbb{R} \\ "by continuity" (i.e. representing r as a limit of rational numbers). \\ \\ & x^{r} = r \times r^{-1}, r \in \mathbb{R} \\ \\ \hline \text{Example differentiation rules:} \\ \\ & f(x) = \cos(4x) \\ \\ & f'(x) = -\sin(4x) \cdot 4 \cdot 1 = -4 \sin(4x) \\ \\ \hline \text{More examples in live lecture} \\ \end{cases}$$

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2. Darivatives
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Derivative of exponential function

$$(e^{\times})' = \lim_{h \to 0} \frac{e^{\times + h} - e^{\times}}{h} = e^{\times} \lim_{h \to 0} \frac{e^{h} - 1}{h}$$

 $\frac{e^{\times + h} - e^{\times}}{h} = e^{\times} \lim_{h \to 0} \frac{e^{h} - 1}{h}$
So in short : $(e^{\times})' = e^{\times} \cdot 5 = 5e^{5\times}$

Derivative of inverse function

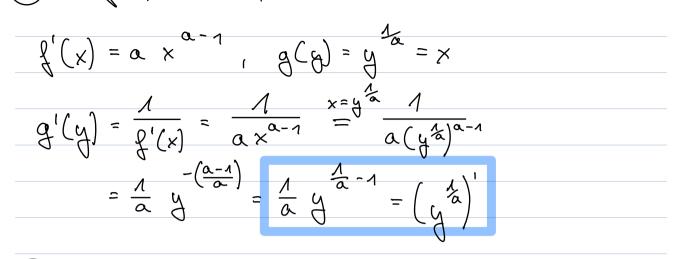
$$f:(a,b) \longrightarrow \mathbb{R}$$
 diff able, invertible on its
range, with inverse $g = f^{-1}$
 $g(f(x)) = x$ (definition of inverse)
Take derivative: $g'(f(x)) \cdot f'(x) = 1$
(chain rule)

Remark: From, e.g., Analysis I

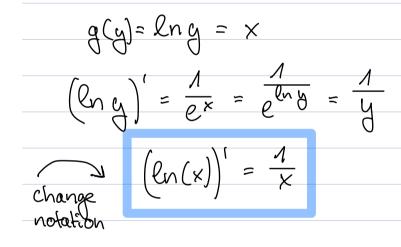
$$f$$
 diffable at x with $f'(x) \neq 0$
 $\Rightarrow g$ diffable at $y = f(x)$

Solve for
$$g'(x)$$
:
 $g'(f(x)) = \frac{1}{f'(x)}$, with $y = f(x)$
 $\Rightarrow \qquad q'(y) = \frac{1}{f'(x)} = \frac{1}{f'(y)}$

(1)
$$f(x) = x^{\alpha}$$
, $\alpha > 0$



(2)
$$f(x) = e^{x}$$
, $f'(x) = e^{x}$



$$3 \qquad f(x) = \tan x = \frac{\sin x}{\cos x} = y$$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot (-\sin x)}{\cos^2(x)}$$

$$guotient$$

$$Full = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$(\operatorname{arctan} y)' = \frac{1}{g'(x)} = \cos^{2} x$$

$$(\operatorname{arctan} y)' = \frac{1}{g'(x)} = \cos^{2} x$$

$$(\operatorname{arctan} y)' = y^{2} = \frac{\sin^{2} x}{\cos^{2} x} = \frac{1 - \cos^{2} x}{\cos^{2} x}$$

$$= \sum \cos^{2} x \cdot y^{2} = 1 - \cos^{2} x$$

$$= \sum \cos^{2} x (1 + y^{2})' = 1$$

$$= \sum \cos^{2} x = \frac{1}{1 + y^{2}}$$

$$= \sum (\operatorname{arctan} y)' = \frac{1}{1 + y^{2}}$$