Week 1: PreFunctions

1. MULTI Single

Find the (complex) roots of the polynomial

$$p(x) = x^2 + 4\,x + 13$$

(a) $x_1 = +2 + 3i$, $x_2 = +2 - 3i$
(b) $x_1 = -3 + 2i$, $x_2 = -3 - 2i$
(c) $x_1 = +3 - 2i$, $x_2 = +3 + 2i$
(d) $x_1 = -2 - 3i$, $x_2 = -2 + 3i$

2. MULTI Single

Let $p(x)$ be a polynomial of degree n with arbitrary complex coefficients. Which of the following is true?

(a) If $p(x) = c(x - \alpha_1)(x - \alpha_2)...(x - \alpha_n)$ with $\alpha_1, ..., \alpha_n \in \mathbb{R}$, then the roots of $p(x)$ can be real and also imaginary.
(b) $p(x)$ has exactly n roots (considering multiplicities)
(c) $p(x)$ can have no roots
(d) If z is a root, then its complex conjugate is z^* is also a root

3. MULTI Single

Find all the values of the parameter λ for which the equation

$$2\,x^2 - \lambda\,x + \lambda = 0$$

has no real solutions.

(a) $\lambda \in \{0, 8\}$
(b) $\lambda \in (0, 8)$
(c) $\lambda \in (-\infty, 0) \cup (8, \infty)$
(d) $\lambda \in (-8, 0)$

4. MULTI Multiple

The number $5.21\overline{37}$ is:

(a) an integer
(b) a real number
(c) a rational number
(d) a natural number

5. MULTI Single

Assuming that $z = a + bi$ is a complex number, compute real and imaginary part of $\dfrac{1}{z^2}$

(a) $\mathrm{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 - b^2}{(a^2 - b^2)^2}, \mathrm{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{2ab}{(a^2 - b^2)^2}$
(b) $\mathrm{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 - b^2}{(a^2 + b^2)^2}, \mathrm{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{-2ab}{(a^2 + b^2)^2}$

(c) $\text{Re}\left(\frac{1}{z^2}\right) = \frac{a^2+b^2}{(a^2+b^2)^2}, \text{Im}\left(\frac{1}{z^2}\right) = \frac{-2ab}{(a^2+b^2)^2}$
(d) $\text{Re}\left(\frac{1}{z^2}\right) = \frac{a^2+b^2}{(a^2+b^2)^2}, \text{Im}\left(\frac{1}{z^2}\right) = \frac{2ab}{(a^2+b^2)^2}$

6. MULTI Single

Consider v, $w \in \mathbb{C}$. Which of the following is NOT true?

(a) $(v^*)^m + (w^*)^n = (v^m + w^n)^*$ for $m, n \in \mathbb{N}$
(b) $v^* + w^* = (v + w)^*$
(c) $v^{-1} = \frac{(v^*)^{-1}}{|v|^2}$ (with $|v| := \sqrt{v \cdot v^*}$)
(d) $v^* \cdot w^* = (v \cdot w)^*$

7. MULTI Single

Let $p(x)$ be a polynomial of degree n with **real** coefficients. Which of the following is true?

(a) $p(x)$ can have less than n complex roots
(b) If $p(x)$ is odd, it can have no roots
(c) $p(x)$ has n distinct real roots
(d) If z is a root, then its complex conjugate is z^* is also a root

8. MULTI Single

Compute $\left|\frac{1+i}{2-i}\right|$.

(a) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{3}}$
(b) $\left|\frac{1+i}{2-i}\right| = \frac{2}{5}$
(c) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{5}}$
(d) $\left|\frac{1+i}{2-i}\right| = \frac{2}{3}$

9. MULTI Single

Which of the following is equal to $\sqrt{i}$?

(a) $\frac{1-i}{\sqrt{2}}$
(b) i
(c) $\frac{1+i}{\sqrt{2}}$
(d) $1-i$

10. MULTI Single

Which of the following does not describe the rational numbers $\mathbb{Q}$?

(a) $\mathbb{Q} = \left\{ \frac{n}{m} \mid n, m \in \mathbb{N} \right\}$

(b) $\mathbb{Q} = \left\{ \frac{n}{m} \mid n \in \mathbb{Z} \text{ and } m \in \mathbb{N} \right\}$

(c) $\mathbb{Q} = \left\{ \frac{n}{m} \mid n, m \in \mathbb{N} \right\} \cup \left\{ \frac{-n}{m} \mid n, m \in \mathbb{N} \right\} \cup \{0\}$

(d) $\mathbb{Q} = \left\{ \frac{n}{m} \mid n, m \in \mathbb{Z} \text{ and } m \neq 0 \right\}$

Total of marks: 10