Week 12: Matrices

1.

MULTI 1.0 point 0 penalty Single Shuffle

Calculate the matrix product:

$$\begin{bmatrix} 1 & 2 & 9 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} = ?$$

(a) $\begin{bmatrix} -6 & 10 & 8 \\ 2 & 6 & 4 \\ 5 & 9 & 7 \end{bmatrix}$ (100%)

(b) $\begin{bmatrix} -6 & 9 & 8 \\ 2 & 6 & 4 \\ 5 & 9 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} -6 & 10 & 8 \\ 2 & 5 & 4 \\ 5 & 9 & 7 \end{bmatrix}$

(d) $\begin{bmatrix} -6 & 9 & 8 \\ 2 & 5 & 4 \\ 5 & 9 & 8 \end{bmatrix}$

2.

MULTI 1.0 point 0 penalty Single Shuffle

Let

$$\mathcal{R} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Which is the inverse of $\mathcal{R}$

(a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (100%)

(b) $\begin{bmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

(c) $\begin{bmatrix} -\cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

(d) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Recall that $A^{-1} \cdot A = \mathrm{id}$ *with* id *being the identity matrix. We have that*

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} =$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \cos\theta\sin\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3.

MULTI 1.0 point 0 penalty Single Shuffle

Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} C = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}$$

Calculate $A \cdot B \cdot C$

(a) $\begin{bmatrix} 12 & 18 \\ 12 & 18 \end{bmatrix}$ (100%)

(b) $\begin{bmatrix} 24 & 24 \\ 2 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 6 & 6 \\ 12 & 18 \end{bmatrix}$

$$A \cdot B \cdot C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 6 \\ 6 & 12 \end{bmatrix} = \begin{bmatrix} 12 & 18 \\ 12 & 18 \end{bmatrix}$$

4.

MULTI 1.0 point 0 penalty Single Shuffle

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 99 & 0 \\ 99 & 99 \\ 99 & 0 \\ 99 & 99 \end{bmatrix} C = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$$

Which of the following is a valid matrix multiplication?

(a) $B \cdot A^T \cdot C$ (100%)
(b) $A \cdot B \cdot C$
(c) $A^T \cdot B^T \cdot C$
(d) $C^T \cdot B^T \cdot A^T$

We can simply analyse the dimensions of the matrices. Then $B \cdot A^T \cdot C$ reads

$$(5 \times 2) \cdot (3 \times 2)^T \cdot (3 \times 1) = (5 \times 2) \cdot (2 \times 3) \cdot (3 \times 1)$$

And thus the dimensions match and the output is a (5×1) matrix

5.

MULTI 1.0 point 0 penalty Single Shuffle

Which of the following is equivalent to $(A \cdot B \cdot C)^T$

(a) $C^T \cdot B^T \cdot A^T$ (100%)
(b) $B^T \cdot C^T \cdot A^T$
(c) $A^T \cdot B^T \cdot C^T$
(d) $C^T \cdot B^T \cdot A^T$

$$(A \cdot B \cdot C)^T = (A \cdot (B \cdot C))^T = (B \cdot C)^T \cdot A^T = C^T \cdot B^T \cdot A^T$$

6.

MULTI 1.0 point 0 penalty Single Shuffle

Let A be a (3×4) matrix, and B be a matrix such that $A^T \cdot B$ and $B \cdot A^T$ are both defined. What are the dimensions of B

(a) (3×4) (100%)
(b) (3×3)
(c) (4×4)
(d) (4×3)

If A is (3×4), then A^T is (4×3). Then if B is (3×4), $A^T \cdot B$ and $B \cdot A^T$ are well defined.

7.

MULTI 1.0 point 0 penalty Single Shuffle

Solve the following system of linear equations:

$$\begin{aligned} x_1 + 3\,x_2 - 5\,x_3 &= 4 \\ x_1 + 4\,x_2 - 8\,x_3 &= 7 \\ -3\,x_1 - 7\,x_2 + 9\,x_3 &= -6 \end{aligned}$$

(a)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}$$

(100%)

(b)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

(d)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Write out the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{array}\right] \xrightarrow[R3+3R1\to R3]{R2-R1\to R2} \left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 2 & -6 & 6 \end{array}\right] \xrightarrow[R3-2R2\to R3]{R1-3R2\to R1} \left[\begin{array}{ccc|c} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow x = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}$$

8.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Find α such that following system of linear equations has no solutions:

$$\begin{aligned} x_1 + \alpha\, x_2 &= 1 \\ x_1 - x_2 + 3\, x_3 &= -1 \\ 2\, x_1 - 2\, x_2 + \alpha\, x_3 &= -2 \end{aligned}$$

(a) $\alpha = -1$ (100%)
(b) $\alpha = 6$
(c) $\alpha = 3$
(d) $\alpha = 1$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & \alpha & 0 & 1 \\ 1 & -1 & 3 & -1 \\ 2 & -2 & \alpha & -2 \end{array}\right] \xrightarrow[\substack{R1-R2\to R2 \\ R3-2R1\to R3}]{R2\to R1} \left[\begin{array}{ccc|c} 1 & -1 & 3 & -1 \\ 0 & \alpha+1 & -3 & 2 \\ 0 & 0 & \alpha-6 & 0 \end{array}\right]$$

The matrix is full rank unless $\alpha = -1$ *or* 6. *When* $\alpha = -1$ *the last 2 equations are* $-3x_2 = 2$ *and* $-7x_3 = 0 \Rightarrow$ *the system has no solutions. When* $\alpha = 6$ *the augmented matrix becomes:*

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & -1 \\ 0 & 7 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow$ *the solution is not unique*

9.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Which of the following is true for Homogeneous systems of Linear Equations?

(a) If $\vec{a}$ and $\vec{b}$ are both solutions, then $\vec{a} + \vec{b}$ is also a solution (100%)
(b) The system might not have a solution
(c) If $\vec{a}$ is a solution, $\exists k \in \mathbb{R}$ such that $k\vec{a}$ is not a solution
(d) We can always find a solution $\vec{a}$ such that all its components a_i are positive

If we have $\vec{a}$ and $\vec{b}$ be solutions of $A\vec{x} = 0$, then we have

$$A(\vec{a} + \vec{b}) = A\vec{a} + A\vec{b} = 0 + 0 = 0$$

Where the first equality comes from the linearity of A

10.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Let $\vec{a}$ and $\vec{b}$ be the solution to a system of linear equations $A\vec{x} = \vec{v}$. When is $\vec{a} + \vec{b}$ also a solution?

(a) When $\vec{v} = 0$ (100%)
(b) When $\vec{v} \neq 0$
(c) Always
(d) Never

$$A(\vec{a} + \vec{b}) = A\vec{a} + A\vec{b} = 2\vec{v}$$

And $2\vec{v} = \vec{v} \Rightarrow \vec{v} = 0$

Total of marks: 10