Week 1: PreFunctions

1.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Find the (complex) roots of the polynomial

$$p(x) = x^2 + 4\,x + 13$$

(a) $x_1 = -2 - 3i$, $x_2 = -2 + 3i$ (100%)
(b) $x_1 = -3 + 2i$, $x_2 = -3 - 2i$
(c) $x_1 = +2 + 3i$, $x_2 = +2 - 3i$
(d) $x_1 = +3 - 2i$, $x_2 = +3 + 2i$

Quadratic formula:

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 13}}{2} = \frac{-4 \pm \sqrt{-36}}{2} = -2 \pm 3i$$

2.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Let $p(x)$ be a polynomial of degree n with arbitrary complex coefficients. Which of the following is true?

(a) $p(x)$ has exactly n roots (considering multiplicities) (100%)
(b) If z is a root, then its complex conjugate is z^* is also a root
(c) If $p(x) = c(x - \alpha_1)(x - \alpha_2)...(x - \alpha_n)$ with $\alpha_1, ..., \alpha_n \in \mathbb{R}$, then the roots of $p(x)$ can be real and also imaginary.
(d) $p(x)$ can have no roots

Consider the fundamental theorem of algebra: "Any polynomial of degree n with complex coefficients is the product of n linear factors." (These factors being the roots.) And roots come in complex conjugate pairs as proven in class.

3.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Find all the values of the parameter λ for which the equation

$$2\,x^2 - \lambda\,x + \lambda = 0$$

has no real solutions.

(a) $\lambda \in (0, 8)$ (100%)
(b) $\lambda \in (-\infty, 0) \cup (8, \infty)$
(c) $\lambda \in \{0, 8\}$
(d) $\lambda \in (-8, 0)$

Check the discriminant: $\Delta = \lambda^2 - 4 \cdot 2 \cdot \lambda = \lambda(\lambda - 8)$

- $\Delta = 0$ *for* $\lambda = 0$ *or* $\lambda = 8$ *(1 real solution)*
- $\Delta > 0$ *for* $\lambda > 8$ *or* $\lambda < 0$ *(2 real solutions)*
- $\Delta < 0$ *for* $\lambda \in (0, 8)$ *(pair of complex-conjugate roots)*

4.

MULTI | 1.0 point | 0 penalty | Multiple | Shuffle

The number $5.21\overline{37}$ is:

(a) a rational number (50%)
(b) a natural number (−50%)
(c) an integer (−50%)
(d) a real number (50%)

Let $x = 0.3737... \implies 99x = 100x - x = 37.37... - 0.37... = 37 \implies x = \frac{37}{99}$
Now note that:
$5.21\overline{37} = 5 + 0.2 + 0.01 + 0.00\overline{37} = 5 + \frac{2}{10} + \frac{1}{100} + \frac{1}{100}\frac{37}{99} = 5\frac{529}{2475}$
which is indeed a fraction.

5.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Assuming that $z = a + bi$ is a complex number, compute real and imaginary part of $\frac{1}{z^2}$

(a) $\text{Re}\left(\frac{1}{z^2}\right) = \frac{a^2 - b^2}{(a^2 + b^2)^2}, \text{Im}\left(\frac{1}{z^2}\right) = \frac{-2ab}{(a^2 + b^2)^2}$ (100%)
(b) $\text{Re}\left(\frac{1}{z^2}\right) = \frac{a^2 + b^2}{(a^2 + b^2)^2}, \text{Im}\left(\frac{1}{z^2}\right) = \frac{-2ab}{(a^2 + b^2)^2}$
(c) $\text{Re}\left(\frac{1}{z^2}\right) = \frac{a^2 - b^2}{(a^2 - b^2)^2}, \text{Im}\left(\frac{1}{z^2}\right) = \frac{2ab}{(a^2 - b^2)^2}$
(d) $\text{Re}\left(\frac{1}{z^2}\right) = \frac{a^2 + b^2}{(a^2 + b^2)^2}, \text{Im}\left(\frac{1}{z^2}\right) = \frac{2ab}{(a^2 + b^2)^2}$

$$\frac{1}{z^2} = \frac{1}{a^2 + 2abi - b^2} = \frac{a^2 - 2abi - b^2}{(a^2 + 2abi - b^2)(a^2 - 2abi - b^2)} = \frac{a^2 - b^2}{(a^2 + b^2)^2} + i\frac{(-2ab)}{(a^2 + b^2)^2}$$

6.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Consider v, $w \in \mathbb{C}$. Which of the following is NOT true?

(a) $v^{-1} = \frac{(v^*)^{-1}}{|v|^2}$ (with $|v| := \sqrt{v \cdot v^*}$) (100%)
(b) $v^* \cdot w^* = (v \cdot w)^*$

(c) $v^* + w^* = (v + w)^*$
(d) $(v^*)^m + (w^*)^n = (v^m + w^n)^*$ for $m, n \in \mathbb{N}$

$$|v|^2 = v \cdot v^* \implies v^{-1} = \frac{v^*}{|v|^2}$$

7.

MULTI 1.0 point 0 penalty Single Shuffle

Let $p(x)$ be a polynomial of degree n with **real** coefficients. Which of the following is true?

(a) If z is a root, then its complex conjugate is z^* is also a root (100%)
(b) $p(x)$ has n distinct real roots
(c) If $p(x)$ is odd, it can have no roots
(d) $p(x)$ can have less than n complex roots

It can be easily verified that $p(z^*) = (p(z))^*$ if the coefficients are real (using the identities from exercise (6)).
Then $p(z) = 0 \implies (p(z))^* = 0 \implies p(z^*) = 0,$

8.

MULTI 1.0 point 0 penalty Single Shuffle

Compute $\left|\frac{1+i}{2-i}\right|$.

(a) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{5}}$ (100%)
(b) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{3}}$
(c) $\left|\frac{1+i}{2-i}\right| = \frac{2}{5}$
(d) $\left|\frac{1+i}{2-i}\right| = \frac{2}{3}$

$$\left|\frac{1+i}{2-i}\right|^2 = \frac{(1+i)(1-i)}{(2-i)(2+i)} = \frac{1+1}{4+1} = \frac{2}{5} \Rightarrow \left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{5}}$$

9.

MULTI 1.0 point 0 penalty Single Shuffle

Which of the following is equal to $\sqrt{i}$?

(a) $\frac{1+i}{\sqrt{2}}$ (100%)
(b) $1 - i$
(c) i

(d) $\dfrac{1-i}{\sqrt{2}}$

$$\left(\frac{1+i}{\sqrt{2}}\right)^2 = \frac{1^2+2i+i^2}{2} = \frac{1+2i-1}{2} = i \implies \sqrt{i} = \frac{1+i}{\sqrt{2}}$$

10.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Which of the following does not describe the rational numbers $\mathbb{Q}$?

(a) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{N}\right\}$ (100%)

(b) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{Z} \text{ and } m \neq 0\right\}$

(c) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{N}\right\} \cup \left\{\frac{-n}{m} \mid n, m \in \mathbb{N}\right\} \cup \{0\}$

(d) $\mathbb{Q} = \left\{\frac{n}{m} \mid n \in \mathbb{Z} \text{ and } m \in \mathbb{N}\right\}$

This set would only describe the positive rational numbers.

Total of marks: 10