Week 2: Functions

1. MULTI Single

For which x is $x^2 - 5x + 6 \geq 0$?

(a) $x \leq -6$ and $x \geq 1$
(b) $2 \leq x \leq 3$
(c) $-6 \leq x \leq 1$
(d) $x \leq 2$ and $x \geq 3$

2. MULTI Single

Solve $5x^2 = x^4 - 14$.

(a) $x_{1,2} = \pm\sqrt{2}$, $x_{3,4} = \pm i\sqrt{7}$
(b) $x_{1,2} = \pm\sqrt{7}$, $x_{3,4} = \pm i\sqrt{2}$
(c) $x_{1,2} = \pm\sqrt{2}$, $x_{3,4} = \pm\sqrt{7}$
(d) $x_{1,2} = \pm\sqrt{7}$, $x_{3,4} = \pm\sqrt{2}$

3. MULTI Single

Solve $x^4 + 4x^2 = 20x + x^3 - 16$ (*Hint:* $x = 2$ satisfies the equation).

(a) $x_{1,2,3,4} = \left\{1, 1, -1 - i\sqrt{7}, -1 + i\sqrt{7}\right\}$
(b) $x_{1,2,3,4} = \left\{1, 2, -1 - i\sqrt{7}, -1 + i\sqrt{7}\right\}$
(c) $x_{1,2,3,4} = \left\{1, 2, +1 - i\sqrt{7}, +1 + i\sqrt{7}\right\}$
(d) $x_{1,2,3,4} = \left\{2, 2, -1 - i\sqrt{7}, -1 + i\sqrt{7}\right\}$

4. MULTI Single

Which function has the roots: $x_{1,2,3,4} = \{-4, 5, 7, 1\}$?

(a) $f(x) = x^4 + 7x^3 - 21x^2 - 167x - 140$
(b) $f(x) = x^4 + 17x^3 + 99x^2 + 223x + 140$
(c) $f(x) = x^4 - 9x^3 - 5x^2 + 153x - 140$
(d) $f(x) = x^4 + 5x^3 - 33x^2 - 113x + 140$

5. MULTI Single

Given the following function: $f(x) = x^3 + bx^2 + cx - 14$ with $b, c \in \mathbb{R}$ and knowing that $f(1 + i) = 0$, solve $f(x) = 0$.

(a) $x_{1,2,3,4} = \{1 + i, 1 - i, \pm 7\}$
(b) $x_{1,2,3} = \{1 + i, 1 - i, 7\}$
(c) $x_{1,2,3,4} = \{1 + i, 1 - i, -1 - i, -1 + i\}$
(d) $x_{1,2,3} = \{1 + i, -1 + i, -7\}$

6. MULTI Single

Find all the roots (real or complex) of the polynomial

$$p(x) = 24 - 8\,x - 18\,x^2 + 18\,x^3 - x^4 - 4\,x^5 + x^6\,.$$

Hint: $x = 3$ is a root. Divide out the associated linear factor and continue with more roots that are easy to guess.

(a) $x_{1,2,3,4,5,6} = \{-2, -1, 1, 3, 1+i, 1-i\}$
(b) $x_{1,2,3,4,5,6} = \{-2, -1, 1, 3, 2+i, 2-i\}$
(c) $x_{1,2,3,4,5,6} = \{-2, -1, 2, 3, 1+i, 1-i\}$
(d) $x_{1,2,3,4,5,6} = \{-3, -1, 2, 3, 1+i, 1-i\}$

7. MULTI Single

Let $f(x) = e^{-9x+3}$. Determine the Domain and Range of $f(x)$ and its inverse$f^{-1}(x)$.

(a) $Dom(f) = [0, \infty)$, $Ran(f) = [0, \infty)$,
$Dom(f^{-1}) = [0, \infty)$, $Ran(f^{-1}) = [0, \infty)$
(b) $Dom(f) = (0, \infty)$, $Range(f) = (-\infty, \infty)$,
$Dom(f^{-1}) = (-\infty, \infty)$, $Ran(f^{-1}) = (0, \infty)$
(c) $Dom(f) = (-\infty, \infty)$, $Ran(f) = [0, \infty)$,
$Dom(f^{-1}) = [0, \infty)$, $Ran(f^{-1}) = (-\infty, \infty)$
(d) $Dom(f) = (-\infty, \infty)$, $Ran(f) = (0, \infty)$,
$Dom(f^{-1}) = (0, \infty)$, $Ran(f^{-1}) = (-\infty, \infty)$

8. MULTI Single

Let $g(x) = e^{-13x^2+7}$. Determine the Domain and Range of $g(x)$.

(a) $Domain(g) = [0, \infty)$, $Range(f) = (0, \infty)$
(b) $Domain(g) = (-\infty, \infty)$, $Range(f) = [0, \infty)$
(c) $Domain(g) = (-\infty, \infty)$, $Range(f) = (0, e^7]$
(d) $Domain(g) = [0, \infty)$, $Range(f) = (0, 1]$

9. MULTI Single

Consider the two variable equation $(y-2)^2 = 2(x-1)$. Which of the following is true?

(a) y does not describe a function of x and x does not describe a function of y
(b) y describes a function of x and x does not describe a function of y
(c) y does not describe a function of x and x describes a function of y
(d) y describes a function of x and x describes a function of y

10. MULTI Single

Find the sum of the binomial coefficients:

$$\sum_{k=0}^{n} \binom{n}{k}$$

(a) 2^n
(b) n^n
(c) $(n!)^2$
(d) e^n

Total of marks: 10