Week 3: Functions and Limits

1. MULTI Single

   Find all horizontal asymptotes of $y(x) = \sin\left(\dfrac{1}{x}\right) \cdot x^2 + \dfrac{1}{x-2}$. (Hint: $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$.)

   (a) $\{y = 2\}$
   (b) $\{y = 0\}$
   (c) $\{y = 0, y = 2\}$
   (d) $\{\}$

2. MULTI Single

   Which of the following is a horizontal asymptote of the function

   $$y = \frac{4x}{\log(|x|^7) + 7x}$$

   (a) $y = 0$
   (b) $y = \dfrac{4}{7}$
   (c) The function has no horizontal asymptote
   (d) $y = \dfrac{4}{7^2}$

3. MULTI Single

   Find all vertical asymptotes of $y(x) = \sin\left(\dfrac{1}{x}\right) \cdot x^2 + \dfrac{1}{x-2}$.

   (a) $\{x = 2\}$
   (b) $\{\}$
   (c) $\{x = 0\}$
   (d) $\{x = 0, x = 2\}$

4. MULTI Single

   Find an oblique asymptote of $y(x) = \sin\left(\dfrac{1}{x}\right) \cdot x^2 + \dfrac{1}{x-2}$. The oblique asymptote $y = ax + b$ can be obtained (if exists) by finding coefficients from the asymptotic behaviour: $a = \lim\limits_{x \to \pm\infty} \dfrac{y(x)}{x}$, $b = \lim\limits_{x \to \pm\infty} y(x) - ax$. (Hint: $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$.)

   (a) $\{y = x\}$
   (b) $\{y = x - 1/2\}$
   (c) $\{y = 2x + 1/2\}$
   (d) $\{y = 2x + 1\}$

5. MULTI Single

   Find all horizontal asymptotes of $y(x) = \dfrac{\left(\ln\left(\frac{1}{x}\right) + \ln(x)\right) \cdot (x^2 + x + 2x) + x + \ln(x)}{x}$

   (a) $\{y = 1\}$
   (b) $\{\}$

(c) $\{y = 0\}$

(d) $\{y = 2\}$

6. [MULTI] [Single]

   Evaluate the limit:
   $$\lim_{v \to 2} \frac{2 - v}{\frac{1}{2} - \frac{1}{v}}$$

   (a) -1

   (b) 2

   (c) 4

   (d) -4

7. [MULTI] [Single]

   Evaluate the limit:
   $$\lim_{y \to 0} \frac{\sqrt{2 + y} - \sqrt{2 - y}}{4y}$$

   (a) 1

   (b) $\sqrt{2}$

   (c) $\dfrac{1}{\sqrt{2}}$

   (d) $\dfrac{1}{4\sqrt{2}}$

8. [MULTI] [Single]

   Evaluate the limit:
   $$\lim_{x \to 0} \left(\sqrt{x} \ln x + e^x x^3 \right).$$

   (a) 0

   (b) $+\infty$

   (c) 1

   (d) $-\infty$

9. [MULTI] [Single]

   Evaluate the limit:
   $$\lim_{x \to 0} \frac{1 - \cos x}{x \tan x}.$$

   (Hint: $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$.)

   (a) 2

   (b) $1/2$

   (c) 1

   (d) 0

10. [MULTI] [Single]

    Let
    $$f(x) := \begin{cases} kx + 7 & \text{for } x \geq 2, \\ x^2 + 19 & \text{for } x < 2. \end{cases}$$

    For what value of k is $\lim_{x \to 2} f(x)$ defined?

(a) 1

(b) 8

(c) 4

(d) 2

Total of marks: 10