Week 4: Limits, Continuity, and Start of Derivatives

1. MULTI Single

Which of the following functions does not have a horizontal asymptote

(a) $f(x) = \dfrac{\log\left(|x|^7\right)}{\log\left(|x|^3\right) + x}$

(b) $f(x) = \dfrac{\log|x|}{x}$

(c) $f(x) = \dfrac{a_0 + a_1 \cdot x + ... + a_n \cdot x^n}{b_0 + b_1 \cdot x + ... + b_n \cdot x^n}$, where all coefficients are non-zero.

(d) $f(x) = \dfrac{e^{|x|}}{x^m + x^{m-1} + ... + x + 1}$ for $m \in \mathbb{N}$

2. MULTI Single

Evaluate the limit:

$$\lim_{x \to 0} \frac{12^x - 1}{x}$$

(a) 12
(b) $\ln(12)$
(c) 0
(d) $1/\ln(12)$

3. MULTI Single

Evaluate the limit:

$$\lim_{N \to \infty} \sum_{i=1}^{N} \frac{i^2}{N^3}$$

(a) 0
(b) 1/3
(c) 1/2
(d) 1

4. MULTI Single

Evaluate the limit:

$$\lim_{N \to \infty} \sum_{i=1}^{N} \frac{1}{i^2 + i}$$

(a) 4/3
(b) 2
(c) 1
(d) 9/8

5. MULTI Single

Check by induction which of the following is true:

(a) $\displaystyle\sum_{k=1}^{n} 2^{k-1} = 2^n + 1$

(b) $\sum_{k=1}^{n}(2k-1) = n^2$
(c) $n! < 2^n$ for $n > 4$
(d) $\sum_{k=1}^{n} k^3 = n^2(n+1)^2$

6. MULTI Single

Let $f(x)$ be a differentiable function. Now consider

$$f_1(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}, \qquad f_2(x_0) = \lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

(a) Neither f_1 or f_2 define the derivative of f
(b) f_1 defines a derivative while f_2 does not
(c) Both f_1 and f_2 define the derivative of f
(d) f_2 defines a derivative while f_1 does not

7. MULTI Single

Using the limit definition of the derivative and letting $f(x) = x^2 + x$, which of the following is true?

(a) $f'(x) = \lim_{h\to 0} \frac{x^2 + 2xh + h + x + h - x^2 - x}{h}$
(b) $f'(x) = \lim_{h\to 0} \frac{x^2 + x + h - x^2 - x}{h}$
(c) $f'(x) = \lim_{h\to 0} \frac{x^2 + h^2 + x + h - x^2 - x}{h}$
(d) $f'(x) = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$

8. MULTI Single

Let $m \geq 2$. Consider the piecewise function $f(x) = \begin{cases} x^m, & \text{if } x < 0 \\ 0, & \text{if } x \geq 0 \end{cases}$
Evaluate $f'(0)$ using the limit definition of the derivative.

(a) $f'(0) = 1$
(b) f is not differentiable at $x = 0$
(c) $f'(0) = 0$
(d) $f'(0) = m - 1$

9. MULTI Single

Consider the piecewise function $f(x) = \begin{cases} -x^2, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ \sin x, & \text{if } x > 0 \end{cases}$
Evaluate $f'(0)$ using the limit definition of the derivative.

(a) $f'(0) = \pi$
(b) $f'(0) = 0$
(c) f is not differentiable at $x = 0$
(d) $f'(0) = 1$

10. MULTI Single

The ReLU (Rectified Linear Unit) function is defined as $\mathrm{ReLU}(x) = \max\{0, x\}$ Which of the following is true?

(a) ReLU is differentiable everywhere but $x = 0$, with $\dfrac{\mathrm{d}}{\mathrm{d}x}\mathrm{ReLU}(x) = \left\{ \begin{array}{ll} 0, & \text{if } x \leq 0 \\ 1, & \text{else} \end{array} \right\}$

(b) ReLU is differentiable at finitely many places

(c) ReLU is nowhere differentiable

(d) ReLU is a differentiable function (differentiable everywhere)

Total of marks: 10