Week 5: Derivatives

1. MULTI Single

   Calculate $\dfrac{\mathrm{d}}{\mathrm{d}t}\left[a^t\right]$ where $a > 0$ is a constant.

   (a) $a^t + t$
   (b) a^t
   (c) $a^t + a$
   (d) $a^t \ln(a)$

2. MULTI Single

   Calculate $\dfrac{\mathrm{d}}{\mathrm{d}t}\left[A\cos\left(\omega t + \varphi\right)\right]$ where A, ω, φ are constants.

   (a) $-A\sin\left(\omega t + \varphi\right)$
   (b) $-A\omega\sin\left(\omega t + \varphi\right)$
   (c) $A\sin\left(\omega t + \varphi\right)$
   (d) $A\omega\sin\left(\omega t + \varphi\right)$

3. MULTI Single

   Given that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$ and $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$, which of the following is true?

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \sinh(x)$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = -\cosh(x)$
   (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \sinh(x)$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \cosh(x)$
   (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = -\sinh(x)$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = -\cosh(x)$
   (d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = -\sinh(x)$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \cosh(x)$

4. MULTI Single

   Calculate $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(a^x + a^{-x}\right)\right]$ where $a > 0$ is a constant.

   (a) $\dfrac{a^x - a^{-x}}{a^x + a^{-x}}$
   (b) $\dfrac{a^x + a^{-x}}{a^x - a^{-x}}$
   (c) $\dfrac{a^x - a^{-x}}{a^x + a^{-x}}\ln a$
   (d) $\dfrac{a^x + a^{-x}}{a^x - a^{-x}}\ln a$

5. MULTI Single

   Calculate $\dfrac{\mathrm{d}^3}{\mathrm{d}x^3}\left[x^4 e^x\right]$.

   (a) $e^x(x^4 + 12x^3 + 24x^2 + 40x)$

(b) $e^x(x^4 + 12x^3 + 36x^2 + 24x)$
(c) $e^x(x^4 + 12x^3 + 24x^2 + 24x)$
(d) $e^x(x^4 + 12x^3 + 36x^2 + 40x)$

6. MULTI Single

Calculate $\dfrac{\mathrm{d}}{\mathrm{d}x}[x^x]$.

(a) $x^x(1 + \ln^3 x)$
(b) $x^x \ln^2 x$
(c) $x^x(1 + \ln x)$
(d) $x^{x-1} + x + 1$

7. MULTI Single

Softplus function is defined as:

$$\text{Softplus}(x) = \ln(1 + e^x)$$

What is the derivative of Softplus and where is it defined?

(a) Softplus is not differentiable
(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\text{Softplus}(x) = \left\{ \begin{array}{ll} 0, & \text{if } x \leq 0 \\ 1, & \text{else} \end{array} \right\}$ and it is defined on $\mathbb{R}\backslash\{0\}$
(c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\text{Softplus}(x) = \dfrac{e^x}{x}$ and is defined on $\mathbb{R}\backslash\{0\}$
(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\text{Softplus}(x) = \dfrac{e^x}{1 + e^x}$ and is defined on $\mathbb{R}$

8. MULTI Single

Choose the expression equivalent to $\displaystyle\sum_{n=0}^{\infty} n \cdot x^n$.

Hint: Recall that $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ and use differentiation.

(a) $\left(\dfrac{1}{1-x}\right)^2$
(b) $-\dfrac{x}{1-x^2}$
(c) $\dfrac{x}{(1-x)^2}$
(d) $\dfrac{x}{1-x}$

9. MULTI Single

Find the derivatives of $e^{3x}\cos 4x$ and $e^{3x}\sin 4x$.

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(4\cos 4x - 3\sin 4x)$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(4\cos 4x + 3\sin 4x)$
(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(3\cos 4x + 4\sin 4x)$ and $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(4\cos 4x - 3\sin 4x)$

(c) $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(3\cos 4x - 4\sin 4x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(4\cos 4x + 3\sin 4x)$

(d) $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(3\cos 4x + 4\sin 4x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(3\cos 4x - 4\sin 4x)$

10. MULTI Single

A fly is trained to fly along $y = x^3$ in such a way that its x coordinate is given by $x(t) = 2t + 1$. What is the value of the y component of the velocity of the fly at time $t = 1$?

(a) $v_y(1) = 54$
(b) $v_y(1) = 6$
(c) $v_y(1) = 68$
(d) $v_y(1) = 2$

Total of marks: 10