# Week 5: Derivatives

1.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Calculate $\frac{\mathrm{d}}{\mathrm{d}t}\left[a^t\right]$ where $a > 0$ is a constant.

(a) $a^t \ln(a)$ (100%)
(b) $a^t$
(c) $a^t + t$
(d) $a^t + a$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[a^t\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[e^{(t\ln(a))}\right] = \underbrace{e^{(t\ln(a))}}_{\textit{outer derivative}} \cdot \underbrace{\ln(a)}_{\textit{inner derivative}} = a^t \ln(a)$$

2.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Calculate $\frac{\mathrm{d}}{\mathrm{d}t}\left[A\cos(\omega t + \varphi)\right]$ where $A, \omega, \varphi$ are constants.

(a) $-A\omega\sin(\omega t + \varphi)$ (100%)
(b) $A\omega\sin(\omega t + \varphi)$
(c) $-A\sin(\omega t + \varphi)$
(d) $A\sin(\omega t + \varphi)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[A\cos(\omega t + \varphi)\right] = A\frac{\mathrm{d}}{\mathrm{d}t}\left[\cos(\omega t + \varphi)\right] = A \cdot \underbrace{\omega}_{\textit{inner derivative}} \cdot \underbrace{-\sin(\omega t + \varphi)}_{\textit{outer derivative}}$$

3.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Given that $\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\sinh(x) = \frac{e^x - e^{-x}}{2}$, which of the following is true?

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \cosh(x)$ (100%)
(b) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = -\sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = -\cosh(x)$
(c) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = -\cosh(x)$
(d) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = -\sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \cosh(x)$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}(e^x + e^{-x}) = \frac{1}{2}(e^x + (-1)e^{-x}) = \sinh(x)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}(e^x - e^{-x}) = \frac{1}{2}(e^x - (-1)e^{-x}) = \cosh(x)$$

4.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(a^x + a^{-x}\right)\right]$ where $a > 0$ is a constant.

(a) $\frac{a^x - a^{-x}}{a^x + a^{-x}} \ln a$ (100%)
(b) $\frac{a^x + a^{-x}}{a^x - a^{-x}} \ln a$
(c) $\frac{a^x - a^{-x}}{a^x + a^{-x}}$
(d) $\frac{a^x + a^{-x}}{a^x - a^{-x}}$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(a^x + a^{-x}\right)\right] =$$

$$\underbrace{\frac{1}{a^x + a^{-x}}}_{\textit{outer derivative of } \ln} \cdot \underbrace{\Big( \overbrace{a^x \ln a}^{\textit{derivative of } a^x = e^{x \ln a}} + \overbrace{a^{-x} \ln a}^{\textit{outer derivative of } a^{-x}} \cdot \overbrace{(-1)}^{\textit{inner derivative of } -x \textit{ (inside } a^{-x})} \Big)}_{\textit{inner derivative of } \ln}$$

$$= \frac{a^x - a^{-x}}{a^x + a^{-x}} \ln a$$

5.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Calculate $\frac{\mathrm{d}^3}{\mathrm{d}x^3}\left[x^4 e^x\right]$.

(a) $e^x(x^4 + 12x^3 + 36x^2 + 24x)$ (100%)
(b) $e^x(x^4 + 12x^3 + 24x^2 + 24x)$
(c) $e^x(x^4 + 12x^3 + 24x^2 + 40x)$
(d) $e^x(x^4 + 12x^3 + 36x^2 + 40x)$

*Use the product rule* $(uv)' = u'v + uv'$:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x^4 e^x\right] = 4x^3 e^x + x^4 e^x = e^x(x^4 + 4x^3)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[x^4 e^x\right] = e^x(x^4 + 4x^3 + 4x^3 + 12x^2) = e^x(x^4 + 8x^3 + 12x^2)$$

$$\frac{\mathrm{d}^3}{\mathrm{d}x^3}\left[x^4 e^x\right] = e^x(x^4 + 8x^3 + 12x^2 + 4x^3 + 24x^2 + 24x) = e^x(x^4 + 12x^3 + 36x^2 + 24x)$$

6.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^x\right]$.

(a) $x^x(1+\ln x)$ (100%)
(b) $x^{x-1}+x+1$
(c) $x^x\ln^2 x$
(d) $x^x(1+\ln^3 x)$

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^x] = \frac{\mathrm{d}}{\mathrm{d}x}\left[e^{x\cdot\ln x}\right] \overset{\textit{chain rule}}{=} \underbrace{x^x}_{\textit{outer}} \cdot \left(\underbrace{\ln(x) + \frac{x}{x}}_{\textit{inner}}\right)$$
$$= x^x(1+\ln x)$$

7.

MULTI 1.0 point 0 penalty Single Shuffle

Softplus function is defined as:

$$\mathrm{Softplus}(x) = \ln(1+e^x)$$

What is the derivative of Softplus and where is it defined?

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{Softplus}(x) = \frac{e^x}{1+e^x}$ and is defined on $\mathbb{R}$ (100%)
(b) $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{Softplus}(x) = \frac{e^x}{x}$ and is defined on $\mathbb{R}\backslash\{0\}$
(c) $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{Softplus}(x) = \left\{\begin{array}{ll} 0, & \text{if } x \leq 0 \\ 1, & \text{else} \end{array}\right\}$ and it is defined on $\mathbb{R}\backslash\{0\}$
(d) Softplus is not differentiable

*Using the chain rule:* $\frac{\mathrm{d}}{\mathrm{d}x}\ln(1+e^x) = \frac{1}{1+e^x}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(1+e^x) = \frac{e^x}{1+e^x}$
*We can also observe that the limits agree for all values of* $x \in \mathbb{R}$

8.

MULTI 1.0 point 0 penalty Single Shuffle

Choose the expression equivalent to $\sum_{n=0}^{\infty} n\cdot x^n$.

*Hint:* Recall that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ and use differentiation.

(a) $\frac{x}{(1-x)^2}$ (100%)
(b) $\left(\frac{1}{1-x}\right)^2$
(c) $-\frac{x}{1-x^2}$
(d) $\frac{x}{1-x}$

*Differentiating the geometric series, we obtain:* $\sum_{n=0}^{\infty} n\cdot x^{n-1} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1-x} = \frac{1}{(1-x)^2}$

9.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Find the derivatives of $e^{3x}\cos 4x$ and $e^{3x}\sin 4x$.

(a) $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(3\cos 4x - 4\sin 4x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(4\cos 4x + 3\sin 4x)$ (100%)
(b) $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(3\cos 4x + 4\sin 4x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(3\cos 4x - 4\sin 4x)$
(c) $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(3\cos 4x + 4\sin 4x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(4\cos 4x - 3\sin 4x)$
(d) $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\cos 4x = e^{3x}(4\cos 4x - 3\sin 4x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}e^{3x}\sin 4x = e^{3x}(4\cos 4x + 3\sin 4x)$

*By direct computation.*

10.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

A fly is trained to fly along $y = x^3$ in such a way that its $x$ coordinate is given by $x(t) = 2t + 1$. What is the value of the $y$ component of the velocity of the fly at time $t = 1$?

(a) $v_y(1) = 54$ (100%)
(b) $v_y(1) = 2$
(c) $v_y(1) = 68$
(d) $v_y(1) = 6$

*The $y$ component of the velocity is given by* $v_y(t) = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x}\cdot\frac{\mathrm{d}x}{\mathrm{d}t} = 3x^2 \cdot 2 = 6(2t+1)^2 \implies v_y(1) = 54$ *with the second equality given by the chain rule.*

*Total of marks: 10*