Week 7: Extreme Values, Integration

1. MULTI Single

What are the maxima and minima of the function $f(x) = \frac{1}{3}x^3 - \frac{7}{2}x^2 + 10x + 3$?

(a) There are no maxima or minima.
(b) At $x = 2$ there is a minimum, at $x = 5$ a maximum.
(c) At $x = 3$ there is a minimum, at $x = 1$ a maximum.
(d) At $x = 2$ there is a maximum, at $x = 5$ a minimum.

2. MULTI Single

For which interval is $f(x) = \frac{x^2}{\pi^2 - x^2}$ positive?

(a) $x \in (-\pi, 0) \cup (\pi, \infty)$
(b) $x \in (-\pi, \pi)$
(c) $x \in (-\infty, -\pi) \cup (\pi, \infty)$
(d) $x \in (-\infty, 0)$

3. MULTI Single

For which values of x does $f(x) = -\ln(x) + \sqrt{x}$ have maxima or minima?

(a) $x = -2$ is a maximum and $x = 2$ is a minimum
(b) $x = 4$ is a minimum
(c) $x = 2$ is a maximum
(d) No maxima or minima

4. MULTI Single

For which value of x does $f(x) = 2\mathrm{e}^{-4/x}$ have a point of inflection?

(a) $x = -4$
(b) There is no point of inflection
(c) $x = 2$
(d) $x = 32$

5. MULTI Single

Evaluate $\int \frac{\cos(\pi/x)}{x^2}\,\mathrm{d}x$. (*Hint*: substitute $\frac{\pi}{x}$.)

(a) $\frac{1}{\pi}\sin\frac{1}{x} + C$
(b) $-\frac{1}{\pi}\sin\frac{\pi}{x} + C$
(c) $-\frac{1}{\pi}\sin\pi x + C$
(d) $\sin\frac{\pi}{x} + C$

6. MULTI Single

Compute $\int \frac{1}{\sqrt{9-x^2}}\,\mathrm{d}x$. *Hint*: How about a substitution involving the sine?

(a) $2\sqrt{9-x^2}+C$
(b) $\sin\left(\frac{x}{3}\right)+C$
(c) $\sin^{-1}\left(\frac{x}{3}\right)+C$
(d) $\cos^{-1}\left(\frac{x}{3}\right)+C$

7. MULTI Single

Evaluate $\int \sin(x)\ln(\cos x)\,\mathrm{d}x$ (*Hint:* use integration by parts)

(a) $\cos x(1+\ln\cos x)+C$
(b) $\cos x(1+\ln\cos x)$
(c) $\cos x(1-\ln\cos x)$
(d) $\cos x(1-\ln\cos x)+C$

8. MULTI Single

Compute $\int x^n e^x \mathrm{d}x$ for $n\in\mathbb{N}$.

(a) $\left(\sum_{k=0}^{n}\frac{n!\,x^{n-k}}{(n-k)!}\right)\mathrm{e}^x$
(b) $\left(\sum_{k=0}^{n}\frac{x^{n-k}}{(n-k)!}\right)\mathrm{e}^x$
(c) $\left(\sum_{k=0}^{n}(-1)^k\frac{n!\,x^{n-k}}{(n-k)!}\right)\mathrm{e}^x$
(d) $\left(\sum_{k=0}^{n}k!\,x^{n-k}\right)\mathrm{e}^x$

9. MULTI Single

Evaluate $\int \sec^2(x)\tan(x)\mathrm{d}x$.

Hint: Try first computing $\frac{\mathrm{d}}{\mathrm{d}x}\sec^2 x$; $\left(\sec(x):=\frac{1}{\cos(x)}\right)$

(a) $\frac{\cos(\sin(x))}{2}+C$
(b) $\frac{1-\tan(x)}{2}+C$
(c) $\frac{\sec^2(x)}{2}+C$
(d) $\frac{\sin^2(x)}{2}+C$

10. MULTI Single

Given $I_n = \int_0^1 (a - bx^3)^n$, find a relationship between I_n and I_{n-1}.

(a) $I_n = \dfrac{3n}{3n+1} I_{n-1}$

(b) $I_n = \dfrac{n}{n+1} I_{n-1}^3$

(c) $I_n = \left(\dfrac{n}{n+1}\right)^3 I_{n-1}$

(d) $I_n = \dfrac{3n}{3n-1} I_{n-1}$

Total of marks: 10