## Week 8: Integrating Rational Functions, and Definite integrals

1. MULTI  Single

   Calculate $\displaystyle\int_0^1 \frac{1}{x^2 + 14x + 98}\,\mathrm{d}x$

   (a) 1
   (b) $\dfrac{1}{28}\left[\pi - \arcsin\left(\dfrac{5}{4}\right)\right]$
   (c) $3\pi$
   (d) $\dfrac{1}{7}\left[\arctan\left(\dfrac{8}{7}\right) - \dfrac{\pi}{4}\right]$

2. MULTI  Single

   Let $P(t)$ denote the number of bacteria in a sample at time $t$. Initially, $P(0) = 100$ and it increases at a rate $\dfrac{\mathrm{d}P}{\mathrm{d}t} = 20e^{3t}$. What is the population at $t = 50$?

   (a) $P(50) \approx 8.4 \times 10^{66}$
   (b) $P(50) \approx 9.3 \times 10^{65}$
   (c) $P(50) \approx 3.5 \times 10^{22}$
   (d) $P(50) \approx 2.8 \times 10^{66}$

3. MULTI  Single

   Calculate $\displaystyle\int_2^1 \frac{2y^3 - 6y^2}{y^2}\,\mathrm{d}t$

   (a) 9
   (b) 3
   (c) $-3$
   (d) $-9$

4. MULTI  Single

   Calculate $\displaystyle\int_0^{1/2} \frac{2x^2 + 2}{x^2 - 1}\,\mathrm{d}x$

   (a) $-1$
   (b) $2\ln(2) - 1$
   (c) $1 - 2\ln(3)$
   (d) $2\ln(3)$

5. MULTI  Single

   Evaluate $\displaystyle\int_0^1 \frac{3x^2 + 12x + 11}{(x + 1)(x + 2)(x + 3)}\,\mathrm{d}x$

   (a) $-\ln(5)$
   (b) $2\ln(2)$
   (c) $\ln(3)$
   (d) $4\ln(2) - 2$

6. MULTI Single

Evaluate $\displaystyle \int \frac{e^x}{e^{2x} - e^x}\,\mathrm{d}x$

(a) $4 - \ln(e^{2x} - e^x) + C$
(b) $-\ln(e^{2x} - e^x)$
(c) $\ln\left|\dfrac{e^x - 1}{e^x}\right| + C$
(d) $\ln\left|\dfrac{e^x - 1}{e^x}\right|$

7. MULTI Single

Evaluate the length of the line $y(x)$ knowing that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{\dfrac{(1 - \cos(x)\sin(x))^2}{\cos^4(x)}e^{-2x} - 1}$

on the interval $x \in [0, 1]$ using the following integral: $\displaystyle L = \int_{p_0}^{p_1} \mathrm{d}s = \int_0^1 \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$.

*Hint:* To compute the resulting integral, look at the integrand carefully and recall the derivative of $\tan x$.

(a) $e^\pi$
(b) $\pi - \tan(0.5)$
(c) $\pi/\tan(e)$
(d) $\tan(1)/e$

8. MULTI Single

Calculate $\displaystyle \int_{-1}^1 f(x)\,\mathrm{d}x$ where

$$f(x) = x\left(\frac{e^x - e^{-x}}{2}\right)\tan(x)$$

(a) $0$
(b) $e$
(c) $-\pi$
(d) $2$

9. MULTI Single

Find $\displaystyle \int_1^e \frac{\ln x}{x}\,\mathrm{d}x$ (*Hint:* use a substitution)

(a) $0.75$
(b) $1$
(c) $0.5$
(d) $1.5$

10. MULTI Single

Calculate $\displaystyle \int_0^{\pi/2} x\sin(x)\cos(x)\,\mathrm{d}x$ (*Hint:* simplify $\sin(x)\cos(x)$ using a trigonometric identity, and then use integration by parts.)

    (a) $\pi/8$
    (b) $3\pi/8$
    (c) $0$
    (d) $\pi/4$

*Total of marks: 10*