Week 9: Application of Integration and Indefinite Integrals

1. MULTI Single

   Find the area A under the curve of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$.

   (a) $A = 2$
   (b) $A = 8$
   (c) $A = \frac{1}{4}$
   (d) $A = \frac{16}{3}$

2. MULTI Single

   Calculate the area between the curves:

   $$y_1(x) = x^2 + 2, \text{ and } y_2 = \sin x,$$

   for values of $x \in (-1, 2)$.

   (a) $A = \frac{7}{3} + 1 + \cos 2 + \cos 1$
   (b) $A = 9 + \cos 2 - \cos 1$
   (c) $A = 9 + \cos 2 + \cos 1$
   (d) $A = \frac{7}{3} + 1 + \cos 2 - \cos 1$

3. MULTI Single

   Calculate the area between $\sin(x)$ and $\cos(x)$ on the interval $[0, 2\pi]$. *Hint*: $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \sin\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}, \cos\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}$.

   (a) $2\sqrt{2}$
   (b) $4\sqrt{2}$
   (c) $\sqrt{2}$
   (d) 0

4. MULTI Single

   The integral $\int_{-\infty}^{\infty} x \, \mathrm{d}x$:

   (a) does not exist
   (b) equals $x^2 + C$
   (c) equals 0
   (d) equals ∞

5. MULTI Single

   Find the area between the curves $x = 1 - y^2$ and $y = -x - 1$.

   (a) 2
   (b) 4.5
   (c) 3.5

(d) 1

6. MULTI Single

Which of the following integrals computes the volume V of a cone of height h and base radius R?

(a) $V = \int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \frac{1}{3}\pi R^2 h$.

(b) $V = \int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \pi \frac{h^2}{R^2} x^2$.

(c) $V = \int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \pi \frac{R^2}{h^2} x^2$.

(d) $V = \int_0^R A(x)\,\mathrm{d}x$ with $A(x) = \pi x^2$.

7. MULTI Single

Compute the (infinite) Taylor series of e^x around $x = 0$. (See the Week 9 Example Session notes.)

(a) $\sum_{n=1}^{\infty} \frac{n \cdot x^n}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{n \cdot x^n}{n!}$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

(d) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

8. MULTI Single

Compute the Taylor series of $\sin(x)$ around $x = 0$. (See the Week 9 Example Session notes.)

(a) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n+1)!}$

(b) $\sum_{n=0}^{\infty} \frac{x^{2n-1}}{(2n-1)!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n-1)!}$

9. MULTI Single

Compute the Taylor series of $\cos(x)$ around $x = 0$. (See the Week 9 Example Session notes.)

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n-1}}{(2n)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n-1)!}$

10. MULTI Single

Evaluate $\displaystyle\int \sqrt{1-x^2}\,\mathrm{d}x$. *Hint:* A trigonometric substitution.

(a) $\dfrac{x\sqrt{1-x^2}}{2} + \dfrac{\arcsin(x)}{2} + C$

(b) $\sqrt{x} + \arctan(x) + C$

(c) $\dfrac{\tan(2x)}{2} + \dfrac{xe^x}{2} + C$

(d) $\dfrac{1}{\sqrt{1-x^2}} + \cos(x) + C$

Total of marks: 10