Week 9: Application of Integration and Indefinite Integrals

1.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Find the area A under the curve of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$.

(a) $A = \frac{16}{3}$ (100%)
(b) $A = 8$
(c) $A = 2$
(d) $A = \frac{1}{4}$

$$A = \int_0^4 \sqrt{x}\,\mathrm{d}x = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3}.$$

2.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Calculate the area between the curves:

$$y_1(x) = x^2 + 2, \text{ and } y_2 = \sin x,$$

for values of $x \in (-1, 2)$.

(a) $A = 9 + \cos 2 - \cos 1$ (100%)
(b) $A = \frac{7}{3} + 1 + \cos 2 - \cos 1$
(c) $A = \frac{7}{3} + 1 + \cos 2 + \cos 1$
(d) $A = 9 + \cos 2 + \cos 1$

$$A = \int_{-1}^{2} (y_1 - y_2)\mathrm{d}x = \int_{-1}^{2} (x^2 + 2 - \sin x)\mathrm{d}x = \frac{x^3}{3} + 2x + \cos x\Big|_{-1}^{2} =$$
$$= \frac{8}{3} + 4 + \cos 2 + \frac{1}{3} + 2 - \cos(-1) = 9 + \cos 2 - \cos 1$$

3.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Calculate the area between $\sin(x)$ and $\cos(x)$ on the interval $[0, 2\pi]$. *Hint*: $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \sin\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}, \cos\left(\frac{5\pi}{4}\right) = \frac{-1}{\sqrt{2}}$.

(a) $4\sqrt{2}$ (100%)
(b) $\sqrt{2}$
(c) $2\sqrt{2}$
(d) 0

Using periodicity of $\sin$ *and* $\cos$ *in the second step, we find*

$$A = \int_0^{2\pi} |\cos(x) - \sin(x)|\,\mathrm{d}x$$

$$= \int_{\pi/4}^{5\pi/4} (\sin(x) - \cos(x))\,\mathrm{d}x + \int_{5\pi/4}^{9\pi/4} (\cos(x) - \sin(x))\,\mathrm{d}x$$

$$= 2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$$

4.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

The integral $\int_{-\infty}^{\infty} x\,\mathrm{d}x$:

(a) equals 0
(b) equals ∞
(c) does not exist (100%)
(d) equals $x^2 + C$

It does not exist according to the definition of an improper integral (see class), since neither $\int_0^{\infty} x\,\mathrm{d}x$ *nor* $\int_{-\infty}^{0} x\,\mathrm{d}x$ *exist.*

5.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Find the area between the curves $x = 1 - y^2$ and $y = -x - 1$.

(a) 4.5 (100%)
(b) 3.5
(c) 2
(d) 1

Find points of intersection:

$$y = -(1 - y^2) - 1 = y^2 - 2 \Rightarrow y^2 - y - 2 = 0 \Rightarrow y_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \{-1, 2\}$$

Then, using y *as an independent variable:*

$$A = \int_{-1}^{2} \Big((1 - y^2) - (-y - 1)\Big)\,\mathrm{d}y = \int_{-1}^{2} (-y^2 + y + 2)\,\mathrm{d}y = -\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y\Big|_{-1}^{2} = \frac{9}{2}$$

6.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Which of the following integrals computes the volume V of a cone of height h and base radius R?

(a) $V = \int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \pi \frac{R^2}{h^2} x^2$. (100%)
(b) $V = \int_0^R A(x)\,\mathrm{d}x$ with $A(x) = \pi x^2$.
(c) $V = \int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \pi \frac{h^2}{R^2} x^2$.
(d) $V = \int_0^h A(x)\,\mathrm{d}x$ with $A(x) = \frac{1}{3}\pi R^2 h$.

The formula $V = \int_0^h A(x)\,\mathrm{d}x$ *with* $A(x) = \pi \frac{R^2}{h^2} x^2$ *is correct similarly to the example discussed in class. (Draw a picture to see this.)*

7.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Compute the (infinite) Taylor series of e^x around $x = 0$. (See the Week 9 Example Session notes.)

(a) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (100%)
(b) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$
(c) $\sum_{n=0}^{\infty} \frac{n \cdot x^n}{n!}$
(d) $\sum_{n=1}^{\infty} \frac{n \cdot x^n}{n!}$

Direct computation:

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n} e^x \Big|_{x=0} = e^0 = 1$$

8.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Compute the Taylor series of $\sin(x)$ around $x = 0$. (See the Week 9 Example Session notes.)

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ (100%)
(b) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n+1)!}$
(c) $\sum_{n=0}^{\infty} \frac{x^{2n-1}}{(2n-1)!}$
(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n-1)!}$

Direct computation.

$$\frac{1}{(2n)!}\frac{\mathrm{d}^{2n}\sin(x)}{\mathrm{d}x^{2n}}\Big|_{x=0} = (-1)^n\frac{\sin(0)}{(2n)!} = 0$$

$$\frac{1}{(2n+1)!}\frac{\mathrm{d}^{2n+1}\sin(x)}{\mathrm{d}x^{2n+1}}\Big|_{x=0} = (-1)^n\frac{\cos(0)}{(2n+1)!} = (-1)^n\frac{1}{(2n+1)!}$$

9.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Compute the Taylor series of $\cos(x)$ around $x = 0$. (See the Week 9 Example Session notes.)

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$ (100%)

(b) $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n-1)!}$

(c) $\sum_{n=0}^{\infty}\frac{x^{2n-1}}{(2n)!}$

(d) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$

Direct computation.

$$\frac{1}{(2n)!}\frac{\mathrm{d}^{2n}\cos(x)}{\mathrm{d}x^{2n}}\Big|_{x=0} = (-1)^n\frac{\cos(0)}{(2n)!} = (-1)^n\frac{1}{(2n)!}$$

$$\frac{1}{(2n+1)!}\frac{\mathrm{d}^{2n+1}\cos(x)}{\mathrm{d}x^{2n+1}}\Big|_{x=0} = (-1)^{n+1}\frac{\sin(0)}{(2n+1)!} = 0$$

10.

MULTI | 1.0 point | 0 penalty | Single | Shuffle

Evaluate $\int\sqrt{1-x^2}\,\mathrm{d}x$. *Hint:* A trigonometric substitution.

(a) $\frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin(x)}{2} + C$ (100%)

(b) $\frac{1}{\sqrt{1-x^2}} + \cos(x) + C$

(c) $\frac{\tan(2x)}{2} + \frac{xe^x}{2} + C$

(d) $\sqrt{x} + \arctan(x) + C$

$$I = \int \sqrt{1-x^2}\,\mathrm{d}x = \int \cos^2(t)\,\mathrm{d}t \textit{ using substitution } x = \sin(t),\ \mathrm{d}x = \cos(t)\,\mathrm{d}t$$

$$I = \int \frac{\cos(2t)+1}{2}\,\mathrm{d}t = \frac{\sin(2t)}{4} + \frac{t}{2} + C = \frac{\sin(t)\cos(t)}{2} + \frac{t}{2} + C =$$

$$= \frac{x\sqrt{1-x^2}}{2} + \frac{\arcsin(x)}{2} + C$$

Total of marks: 10