Jacobs University
Fall 2022

November 29, 2022

Operations Research

Homework 12

Due on December 7, 2022

Note: Your homework must be submitted via moodle (see the link on the class website) on the due day BEFORE THE TUTORIAL, i.e., before 20:45.

Problem 1 [7 points]

(HL, Exercise 18.7-8.) Suppose that the demand D for a spare airplane part has an exponential distribution with mean 50, that is,

$$\varphi_D(x) = \begin{cases} \frac{1}{50} e^{-x/50} & \text{for } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

This airplane will be obsolete in 1 year, so all production of the spare part is to take place at present. The production costs now are \$1 000 per item—that is, $c = 1\,000$—but they become \$10 000 per item if they must be supplied at later dates—that is, $p = 10\,000$. The holding costs, charged on the excess after the end of the period, are \$300 per item.

(a) Determine the optimal number of spare parts to produce.

(b) Suppose that the manufacturer has 23 parts already in inventory (from a similar, but now obsolete airplane). Determine the optimal inventory policy.

(c) Suppose that p cannot be determined now, but the manufacturer wishes to order a quantity so that the probability of a shortage equals 0.1. How many units should be ordered?

(d) If the manufacturer were following an optimal policy that resulted in ordering the quantity found in part (c), what is the implied value of p?

Problem 2 [7 points]

(a) Use the graphical method to maximize

$$Z = x_1 + 2\,x_2$$

subject to

$$x_1^2 + x_2^2 \leq 1\,,$$
$$x_1, x_2 \geq 0\,.$$

(b) Write a Pyomo program to confirm your answer. (Use the `ipopt` solver instead of `glpk`.)

Problem 3 [6 points]

(HL, Exercise 12.2-3.) Consider the variation of the Wyndor Glass Co. example where the second and third functional constraints of the original problem have been replaced by $9x_1^2 + 5x_2^2 \leq 216$. Demonstrate that $(x_1, x_2) = (2, 6)$ with $Z = 36$ is indeed optimal by showing that the objective function line $36 = 3x_1 + 5x_2$ is tangent to this constraint boundary at $(2, 6)$. (Hint: Express x_2 in terms of x_1 on this boundary, and then differentiate this expression with respect to x_1 to find the slope of the boundary.)