Operations Research, Jacobs University, Fall 2022 — Session 13

Prof. Sören Petrat — Oct. 13, 2022

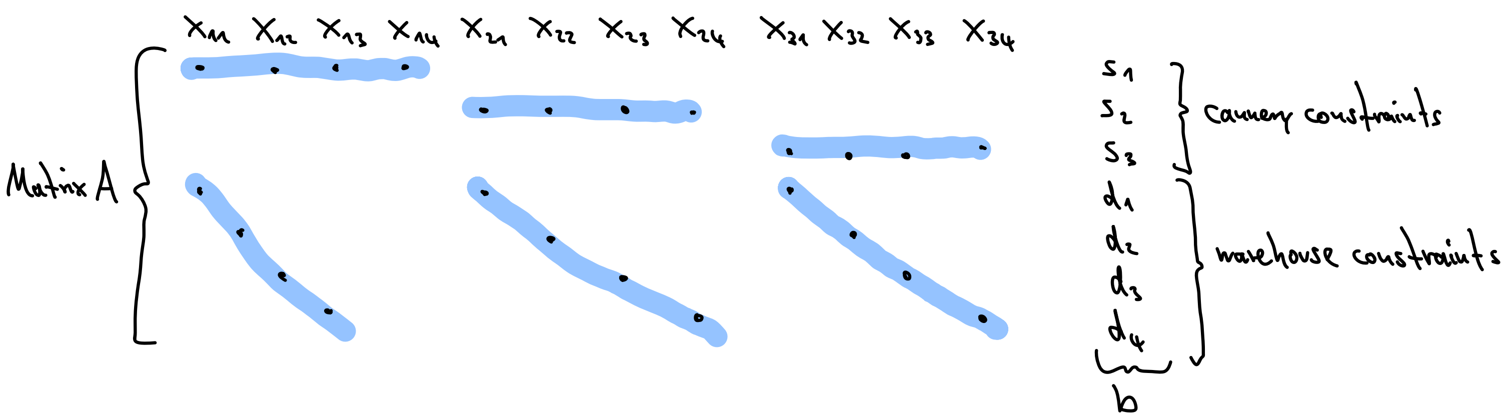
Last time we considered a transportation problem

- Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$ (transportation cost),
- subject to: $\sum_{j=1}^{n} x_{ij} = s_i$ for all $i = 1, \dots, m$,

  $\sum_{i=1}^{m} x_{ij} = d_j$ for all $j = 1, \dots, n$,

  $x_{ij} \geq 0$.

Here, the constraints have a special pattern:



For this type of problem the following holds:

- There are feasible solutions if and only if $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$ (supply = demand)
- If all s_i and d_j have integer values, then all basic variables in all basic feasible solutions have integer values. ← sometimes important for applications
- A streamlined simplex method is available. (We skip the details.)

  ← important for large scale problems

Now: consider additional difficulties

We use the Metro Water District example (Hillier, Lieberman Ch. 8):
↳ Water from 3 rivers needs to be distributed to 4 cities

	Transportation Costs				
	City 1	City 2	City 3	City 4	Supply
River 1	16	13	22	17	50
River 2	14	13	19	15	60
River 3	19	20	23	/	50
Minimum request	30	70	0	10	
Maximum request	50	70	30	∞	

City 4 cannot be supplied with water from River 3.

We have upper and lower bounds for decision variables

Goal: Write this in the standard transportation problem form.

Note:

- Upper bound for City 4 can be replaced by $\underbrace{(50+60+50)}_{\text{total supply}} - \underbrace{(30+70)}_{\text{minimum needed by other cities}} = 60$

- We replace River 3 / City 4 entry by a very large cost M.
  ↳ then every optimal solution will have $x_{34} = 0$

- Problem: requested demand (210) ≥ supply (160)
  We solve this by introducing a "dummy source" with a supply of 50 (= 210 − 160)

This leads to:

	Transportation Costs					
	City 1 (min.)	City 1 (extra)	City 2	City 3	City 4	Supply
River 1	16	16	13	22	17	50
River 2	14	14	13	19	15	60
River 3	19	19	20	23	M	50
Dummy	M	0	M	0	0	50
~~Minimum~~			~~70~~	~~0~~	~~10~~	
Demand	30	20	70	30	60	

50-20

real supply 160, but maximum request by other cities is 50+70+30, so minimum 10 is always guaranteed

no minimum (if all $x_{ij} \geq 0$)

already equality constraint

actual demand shall not receive from dummy source

might receive from dummy source

The simplex method gives us the following result:

	1 min.	1 extra	2	3	4
1			50		
2			20		40
3	30	20			
4 (D)				30	20
	30	20	70	30	60

$Z = 2460$

$\Rightarrow$ Actual water delivered:
- City 1: $30+20=\boxed{50}$
- City 2: $\boxed{70}$
- City 3: $30-30=\boxed{0}$
- City 4: $60-20=\boxed{40}$