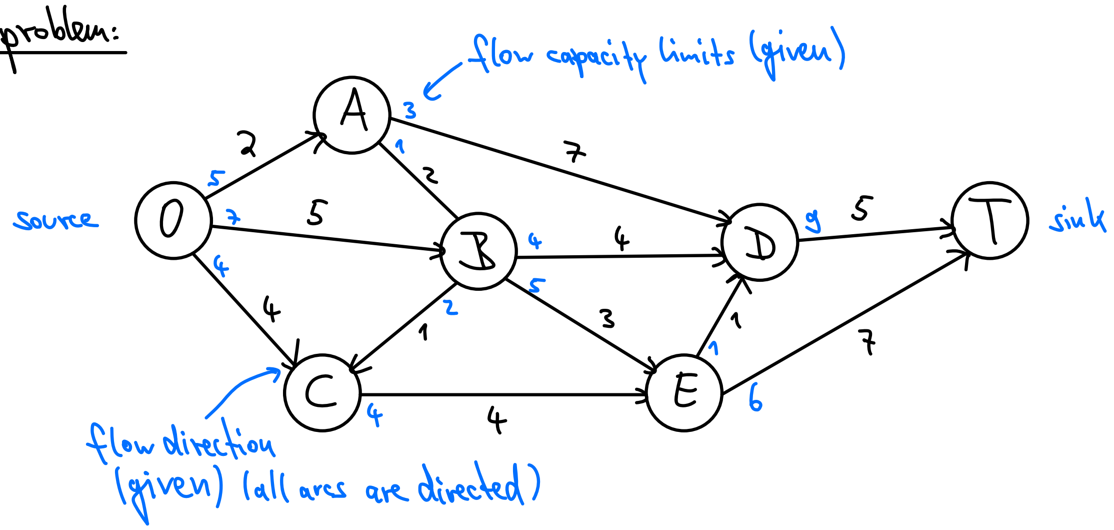
Operations Research, Jacobs University, Fall 2022 Session 16

Prof. Sören Petrat Oct. 25, 2022

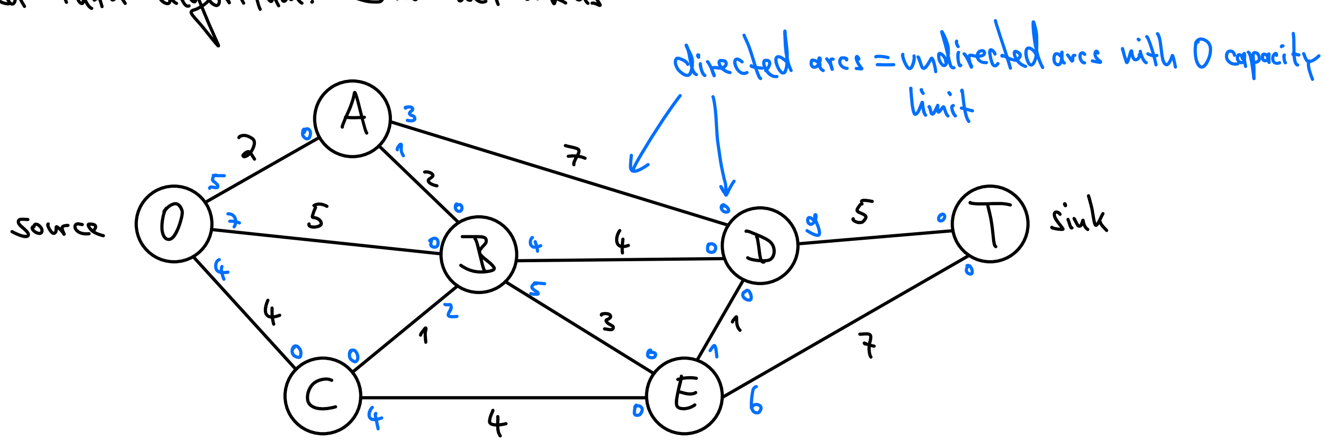
Another problem type is the following:

- <u>Maximum Flow problem:</u>



Objective: maximize flow from source to sink

Augmented Path algorithm: Draw network as



Augmenting path: directed path from source to sink s.t. every arc has strictly positive capacity

Now: • choose an augmenting path

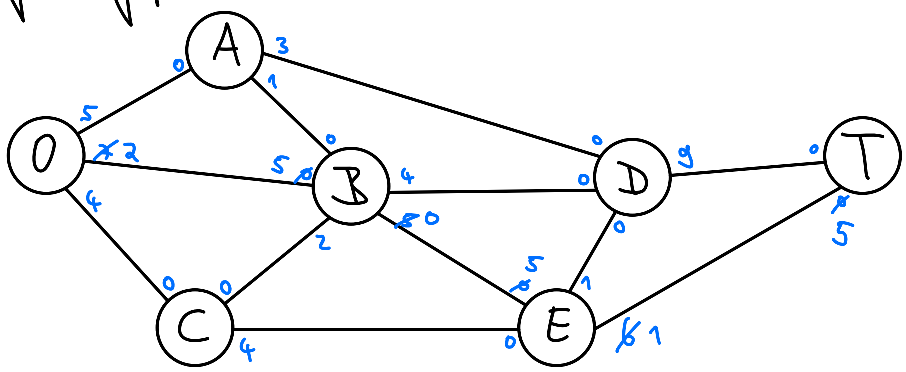
- increase flow by residual capacity = minimum of all capacities along path

  in picture: smallest possible number at beginning of arcs

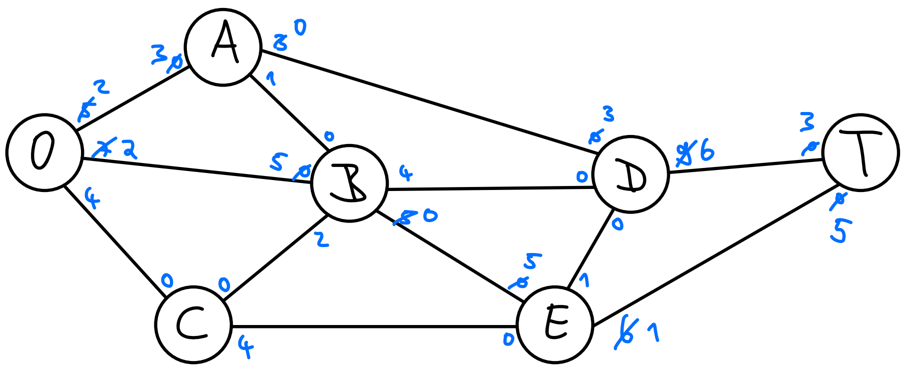
- change capacity limits accordingly
- repeat until no augmenting path can be chosen anymore

For our example:

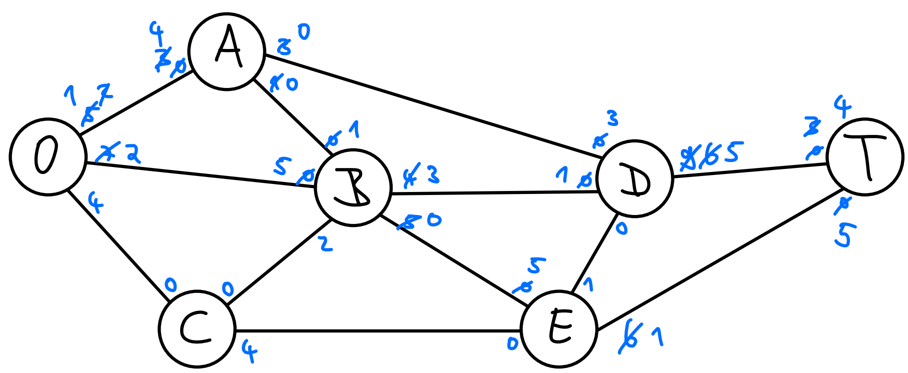
A possible augmenting path is 0-B-E-T: residual capacity: 5 (B-E)



arbitrary next choice: 0-A-D-T: res. cap.: 3 (A-D)



0-A-B-D-T: res. cap.: 1 (A-B)

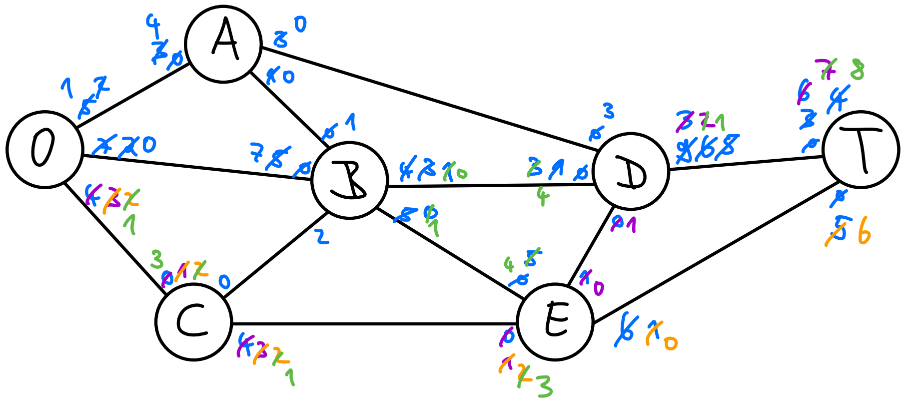


0-B-D-T: res. cap.: 2 (0-B)

0-C-E-D-T: res. cap.: 1 (E-D)

0-C-E-T: res. cap.: 1 (E-T)

0-C-E-B-D-T: res. cap.: 1 (B-D)

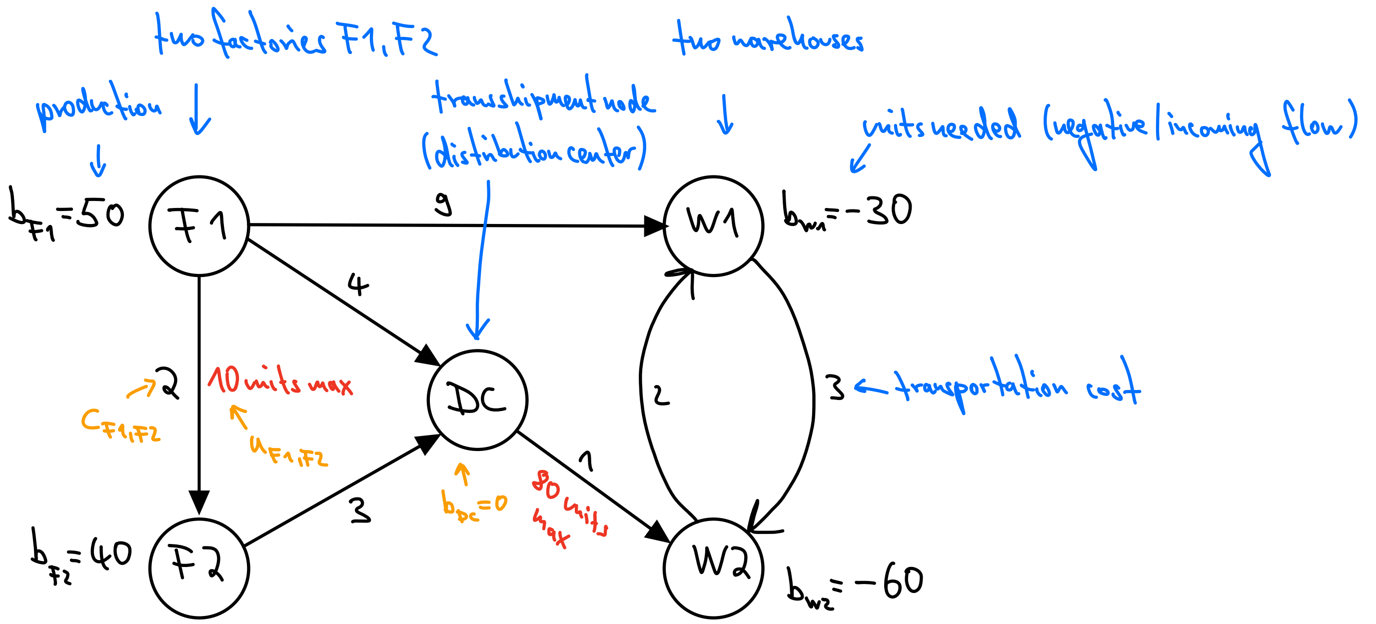


$\Rightarrow$ No more augmenting paths, we have found an optimal solution: $8+6=14$ trips can be made from 0 to T (more details can be read off from final picture).

More generally, all the previous 3 problem types can be formulated as

$\boxed{\text{minimum cost flow problems.}}$

Example (Hillier, Lieberman Chapters 3.4 and 9.6):



Nodes $N=\{F1, F2, DC, W1, W2\}$

General formulation:

- nodes $i \in N$
- directed arcs $(i,j) \in A$
- c_{ij}: unit cost of transportation on arc (i,j)
- u_{ij}: max. capacity on arc (i,j)
- node constraints
  - $b_i > 0$ for supply/source nodes
  - $b_i < 0$ for demand/sink nodes
  - $b_i = 0$ for transshipment nodes
- x_{ij}: flow from i to j (decision variables)