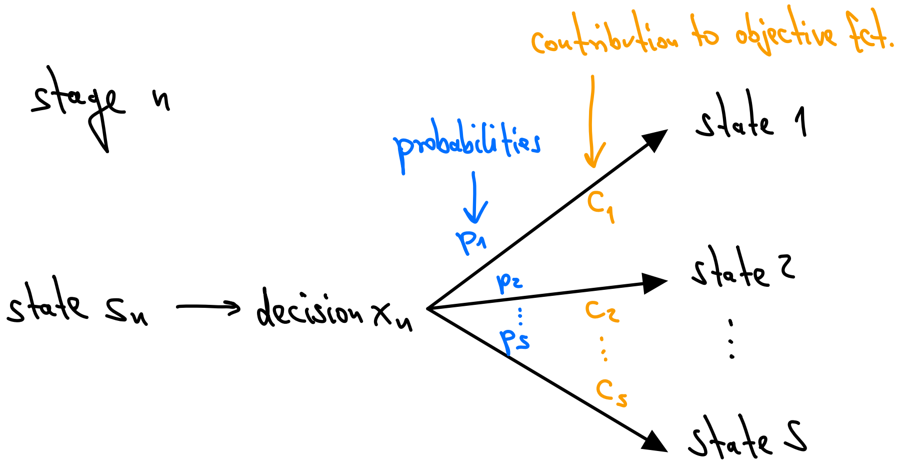
Operations Research, Jacobs University, Fall 2022 — Session 20

Prof. Sören Petrat — Nov. 10, 2022

Next: Optimization problems which involve probability

We start with <u>Probabilistic Dynamic Programming</u>.

Basic structure:



Objective: usually minimize the <u>expected</u> sum of contributions, e.g., costs.

Example: Hit and Miss Manufacturing Co. (Hillier, Lieberman Chapter 10.4)

Setup: • product produced meets strict quality requirements only with probability $p = \frac{1}{2}$

$\Rightarrow$ if x items are produced, the probability for producing only bad items is $\left(\frac{1}{2}\right)^x$ (probability that at least one is good is $1-\left(\frac{1}{2}\right)^x$) (binomial distribution)

(note: extra produced items are called "reject allowance")

• for each new batch there are: • 300 \$ setup costs
  • 100 \$ cost per item

• at most 3 batches can be started, items can be inspected after each batch

• if no good item produced, there is a penalty of 1600 \$

- objective: choose production schedule to minimize costs
- decision variables: x_n = # of items to produce in batch/stage $n = 1,2,3$
- state s = # of acceptable items that still need to be produced = 0 or 1.

done, have produced a good one

no good item yet, might need to continue with next batch

Similar to before, we introduce:

$f_n(s_n, x_n)$ = expected cost for stages n onwards given state s_n, decision x_n, and optimal after

$$f_n^*(s_n) = \min_{x_n = 0,1,2,\dots} f_n(s_n, x_n)$$

Here: $f_n(0, x_n) = 0$ (no new batch is started if good item was already produced)

$$f_n(1, x_n) = \underbrace{K(x_n)}_{\substack{= \begin{cases} 0 & \text{for } x_n = 0 \\ 3 & \text{for } x_n > 0 \end{cases} \\ = \text{setup costs}}} + \underbrace{x_n}_{\text{cost per item}} + \underbrace{\left(\tfrac{1}{2}\right)^{x_n} f_{n+1}^*(1)}_{\substack{\text{expected costs if only} \\ \text{bad items are produced;} \\ \text{we start with } f_4^*(1) = 16}}$$

all costs are in units of 100 $

Solution:

stage/batch $n = 3$:

$f_3(1, x_3) = K(x_3) + x_3 + \left(\tfrac{1}{2}\right)^{x_3} \cdot 16$

s	$x_3 = 0$	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$x_3 = 4$	$x_3 = 5$	...	$f_3^*(s)$	x_3^*
0	0	–	–	–	–	–	...	0	0
1	$0+0+16 = 16$	$3+1+8 = 12$	$3+2+4 = 9$	$3+3+2 = 8$	$3+4+1 = 8$	$3+5+\frac{1}{2} = 8\frac{1}{2}$	... (≥ 9)	8	3 or 4

$n=2$:

s	$f_2(1,x_2) = K(x_2) + x_2 + \left(\frac{1}{2}\right)^{x_2} f_3^*(1)$						f_2^*	x_2^*
	$x_2=0$	$x_2=1$	$x_2=2$	$x_2=3$	$x_2=4$	...		
0	0	–	–	–	–	...	0	0
1	$0+0+8$ $=8$	$3+1+4$ $=8$	$3+2+2$ $=7$	$3+3+1$ $=7$	$3+4+\frac{1}{2}$ $=7\frac{1}{2}$	... (≥ 8)	7	2 or 3

$n=1$:

s	$f_1(1,x_1) = K(x_1) + x_1 + \left(\frac{1}{2}\right)^{x_1} f_2^*(1)$					f_1^*	x_1^*
	$x_1=0$	$x_1=1$	$x_1=2$	$x_1=3$	...		
1	$0+0+7$ $=7$	$3+1+\frac{7}{2}$ $=7\frac{1}{2}$	$3+2+\frac{7}{4}$ $=6\frac{3}{4}$	$3+3+\frac{7}{8}$ $=6\frac{7}{8}$	... (≥ 7)	$6\frac{3}{4}$	2

$\Rightarrow$ Optimal strategy: produce 2 items in first batch; if not successful,
2 or 3 items in second batch; if not successful,
3 or 4 items in third batch.

The associated minimal total expected cost is 675$.