Operations Research, Jacobs University, Fall 2022 Session 21
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3.2 Decision Analysis

We consider decisions to be made where consequences/outcomes are uncertain, e.g.,

- how much of a product sells (demand)
- supply availability
- whether or not to invest in equipment, securities, production facilities, ...

In practice, the following problem often arises:

- Prior probabilities are available for different scenarios, based on past experiences or intuition;
- We can invest in testing or experimentation to reduce uncertainties (= find better probabilities), e.g., test a product in a small market first, or get a more thorough analysis from experts/consultants.
  This costs extra, but often leads to higher expected profits.

Goal: maximize <u>expected</u> profit (or minimize expected costs, etc.)

Guiding example for this chapter: Goferbroke Oil Co. (Hillier, Lieberman: Ch. 15)

Setting: • Company holds land where there might be oil or not (if not, land is "dry").
• Decision: Drill or sell?

Alternatives:	Payoff (in 1000$) in state: Oil	Dry
Drill (costs 100)	$800-100=700$	-100
Sell	90	90
prior probability	$\frac{1}{4}$	$\frac{3}{4}$

Given the prior probabilities, the expected payoffs are $(p=\frac{1}{4})$:

- $\overbrace{\mathbb{E}[\text{Drill}]}^{\text{expected payoff if we drill}} = 700p - 100(1-p) = \frac{700}{4} - \frac{300}{4} = 100$
- $\mathbb{E}[\text{Sell}] = 90p + 90(1-p) = 90$

$\Rightarrow$ Drilling seems preferable here.

Note: For what probability p is it worth drilling?

Want $700p - 100(1-p) \geq 90 \Rightarrow 800p \geq 190 \Rightarrow p \geq \frac{190}{800} \approx 0.24$

This is very close to $\frac{1}{4}$, so maybe some experimentation is advisable.

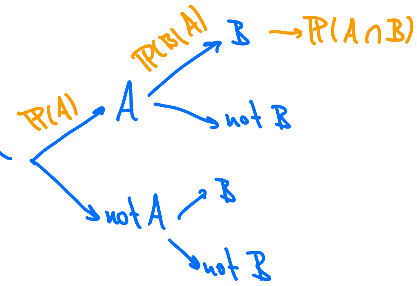
For this example: We can do a seismic survey to find better probabilities:

- cost: 30 (k$)
- probabilities: $\mathbb{P}(\text{Favorable} \mid \text{Oil}) = 0.6$ ($= \mathbb{P}(F \mid \text{Oil})$), $\mathbb{P}(\text{Unfavorable} \mid \text{Oil}) = 0.4$ ($= \mathbb{P}(\text{Unf} \mid \text{Oil})$)
  $\underbrace{\mathbb{P}(\text{Favorable} \mid \text{Dry})}_{\text{probability for favorable outcome if land is dry}} = 0.2$, $\mathbb{P}(\text{Unfavorable} \mid \text{Dry}) = 0.8$

  conditional probabilities

Now we should find probabilities for oil <u>given</u> favorable/unfavorable outcome, i.e.,
$\mathbb{P}(\text{Oil} \mid F)$, $\mathbb{P}(\text{Oil} \mid \text{Unf})$.

Recall: $\underbrace{\mathbb{P}(A \cap B)}_{\text{prob. for } A \text{ and } B} = \underbrace{\mathbb{P}(A \mid B)}_{\text{prob. for } A \text{ given } B} \underbrace{\mathbb{P}(B)}_{\text{prob. for } B} = \mathbb{P}(B \cap A) = \mathbb{P}(B \mid A)\mathbb{P}(A)$



$$\Rightarrow \boxed{\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\mathbb{P}(A)}{\mathbb{P}(B)} \quad (\text{Bayes' rule})}$$

Let us first compute $\mathbb{P}(F)$:

$$\begin{aligned}\mathbb{P}(F) &= \mathbb{P}(F \mid \text{Oil})\,\mathbb{P}(\text{Oil}) + \mathbb{P}(F \mid \text{Dry})\,\mathbb{P}(\text{Dry}) \\ &= 0.6\,\tfrac{1}{4} + 0.2\,\tfrac{3}{4} \\ &= \tfrac{6}{40} + \tfrac{6}{40} = \tfrac{12}{40} = \tfrac{3}{10} = 0.3 \qquad \Rightarrow \mathbb{P}(\text{Unf}) = 0.7\end{aligned}$$

$$\Rightarrow \mathbb{P}(\text{Oil} \mid F) = \frac{\mathbb{P}(F \mid \text{Oil}) \cdot \mathbb{P}(\text{Oil})}{\mathbb{P}(F)} = \frac{0.6 \cdot \frac{1}{4}}{0.3} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$\mathbb{P}(\text{Oil} \mid \text{Unf}) = \frac{0.4 \cdot \frac{1}{4}}{0.7} = \frac{1}{7}$$

We continue the example next time.