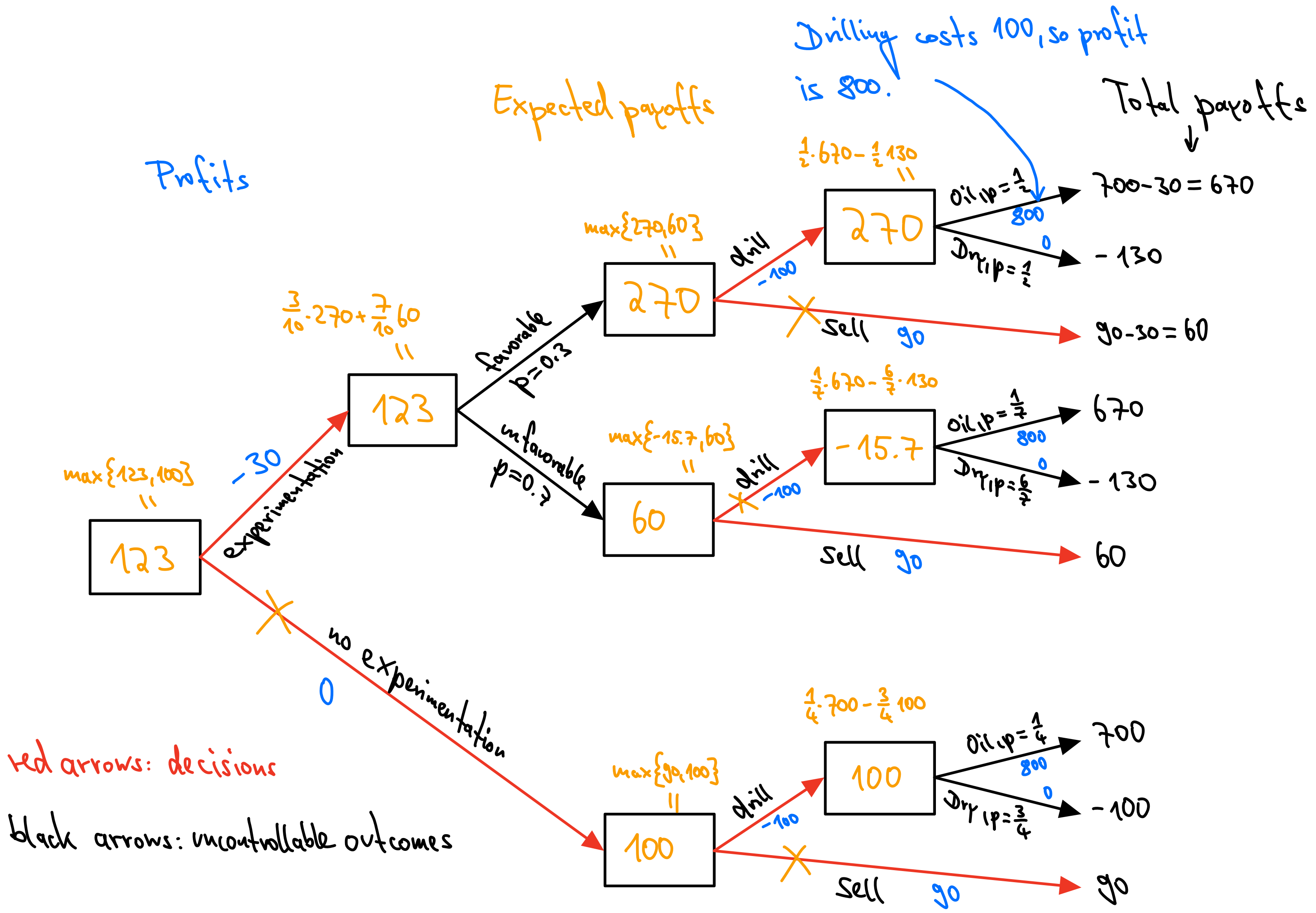
Operations Research, Jacobs University, Fall 2022 Session 22

Prof. Sören Petrat Nov. 17, 2022

We continue our example from last time. Our computations lead us to the following decision tree:



Result:
- Do experimentation
- If favorable: drill
- If unfavorable: sell
- The overall expected profit is 123 000 $.

Note:

- It is often useful to consider the $\boxed{\text{expected value of experimentation (EVE)}}$

  = expected payoff with experimentation − expected payoff without experimentation
  (not including cost of experimentation)

  $= (123+30) - 100 = 53$

  Here, $53 > 30$ (cost of experimentation), so exp. should be done.

  Generally: If EVE > cost of experimentation, then exp. should be done.

- If exp. would lead to perfect outcomes, we should consider the expected value of perfect information $\boxed{\text{(EVPI)}}$

  = expected payoff if state is perfectly known after exp. − expected payoff without exp.

  $= \left(\frac{1}{4} \cdot 700 + \frac{3}{4} 90\right) - 100$

  $= 142.5$ ← if this were less than 30, then exp. would not be advisable,
  so sometimes EVPI can be used to exclude exp.
  (advantage: it is much easier to compute EVPI than EVE)

3.3 Inventory Theory

Inventory management is very important in the business world e.g., for

- retail
- factories (materials for production / resources need to be available)

General ideas:

- Costs for storing ("carrying") and resupplying inventory, but also penalties if not enough inventory available.
- First, we look at deterministic models, where the demand is known (e.g., production). After, we look at stochastic models, where demand is a random variable.

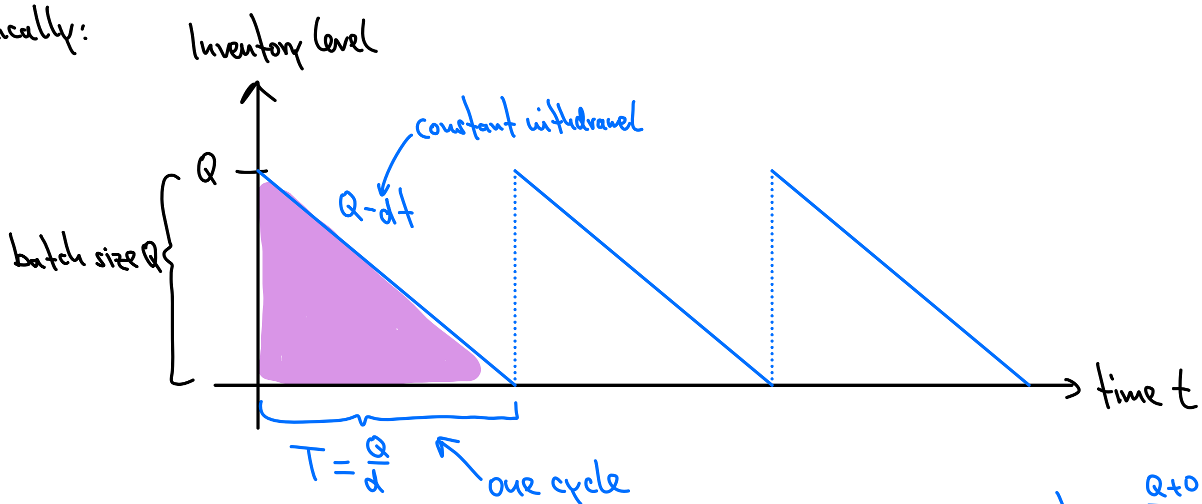
First, let us consider inventory management under the following assumptions:

- The cost of ordering is:
  - K setup costs per order,
  - c unit costs.
- The holding (or storage) cost is h per unit per time in inventory.
- There is a constant withdrawal rate of d units per time.
- We do not allow for shortages.
- There is continuous review, i.e., inventory level is continuously checked (as opposed to periodic checks)

These assumptions lead to the basic economic order quantity model (EOQ).

Note: Under these assumptions, it is always optimal that new orders arrive exactly when inventory is empty.

Graphically:



average inventory $= \frac{Q+0}{2} = \frac{Q}{2}$

The cost per cycle is then $C_{cycle} = \underbrace{K + cQ}_{\text{order costs}} + \underbrace{h \frac{Q}{2} T}_{\text{holding costs}}$

$= h \cdot (\text{average inventory}) \cdot (\text{time})$

$= h \cdot (\text{area of triangle})$

$$\Rightarrow \text{Total cost per time } C = \frac{C_{cycle}}{T} = \frac{K + cQ + h\frac{Q}{2}T}{T} = \frac{K+cQ}{T} + h\frac{Q}{2} = \frac{K+cQ}{Q/d} + h\frac{Q}{2}$$

$$\Rightarrow C = \frac{dK}{Q} + dc + h\frac{Q}{2}$$

What is the optimal order quantity Q^* that minimizes cost per time C?

→ We need to find the minimum:

$$\frac{dC}{dQ} = -\frac{dK}{Q^2} + \frac{h}{2} \overset{!}{=} 0 \Rightarrow \boxed{Q^* = \sqrt{\frac{2dK}{h}} \quad \text{(EOQ formula)}}$$

The corresponding optimal cycle time is $\boxed{T^* = \frac{Q^*}{d} = \sqrt{\frac{2K}{dh}}}$

This basic EOQ model applies to the following Speakers example (Hillier, Liebermann: Chapter 18.1):

- 12 000 \$ setup cost for producing a batch of speakers
- 10 \$ cost for producing one speaker
- 0.30 \$ holding costs per speaker per month (storage space, but also costs of tied up capital)
- speakers are used for continuous production of TVs, $d = 8000$ per month

Then $Q^* = \sqrt{\dfrac{2 \cdot 8000 \cdot 12000}{0.3}} = 25298$ units should be produced every

$T^* = \dfrac{25298}{8000} \approx 3.2$ months.