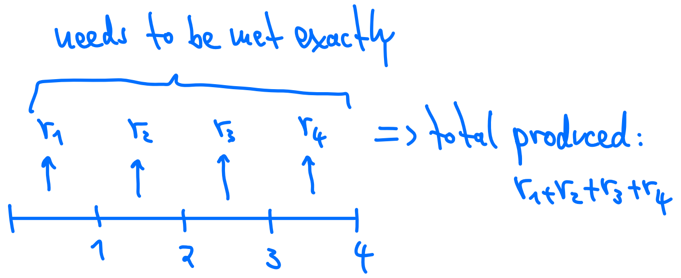
Operations Research, Jacobs University, Fall 2022 — Session 24

Prof. Sören Petrat — Nov. 24, 2022

Next, we consider $\boxed{\text{periodic-review models.}}$



Setup:
- n periods
- r_i = demand in period i
- as before:
  - K = setup cost (for producing/purchasing)
  - c = unit cost of producing/purchasing
  - h = holding cost for each unit left in inventory at end of period

Goal: find optimal production schedule (to minimize cost)

E.g.:
- produce all at beginning: low setup costs, but high holding costs
- produce only when necessary: high setup costs, no holding costs

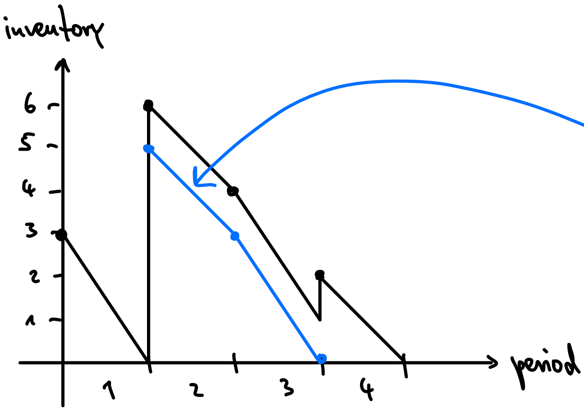
Note: Every production schedule satisfies total demand $\sum_{i=1}^{n} r_i$, so the unit cost c is irrelevant for finding the optimal schedule. We will drop it from now on.

Example: Small airplane manufacturer (Hillier, Liebermann: Chapter 18.4)

- customer orders 10 jets
- delivery in 4 periods: $r_1 = 3$, $r_2 = 2$, $r_3 = 3$, $r_4 = 2$
- setup costs: 2 (million \$)
- holding costs: 0.2 (million \$) per jet per period

Note:

An example of a production schedule is:



	1	2	3	4	
produce:	3	6	0	1	=> total = 10
deliver:	3	2	3	2	=> total = 10
cost	K	K+4h	h	K	=> total cost: 3K+5h

But this cannot be optimal. We should only produce when inventory is empty (otherwise we can find a schedule with same setup costs, but lower holding cost).

Now we can use dynamic programming:

- state s = # of jets in inventory
- x_i = jets produced in period i
- $$f_i(s, x_i) = \begin{cases} hs + f^*_{i+1}(s - r_i + x_i) + K & , s = 0 \quad \leftarrow \text{produce (only when inventory empty, i.e., } s=0) \\ hs + f^*_{i+1}(\underbrace{s - r_i + x_i}_{\text{inventory - demand + produced = \# units in next inventory}}) & , s > 0 \quad \leftarrow \text{don't produce} \end{cases}$$

Solution:

$i=4$: $$f_4^*(s) = \begin{cases} K = 2 & \text{if } s = 0 \\ hs = 0.2 \cdot 2 = 0.4 & \text{if } s = 2 \end{cases}$$

$i=3$:

K=2

h=0.2

	$f_3(s,x_3) = hs + f_4^*(s - r_3 + x_3)$ $(+K \text{ if } s=0)$, $r_3 = 3$				
	$x_3 = 0$	$x_3 = 3$	$x_3 = 5$	f_3^*	x_3^*
$s = 0$	(need to satisfy demand)	$2+2=4$	$0.4+2=2.4$	2.4	5
$s = 3$	$0.2 \cdot 3 + 2 = 2.6$	(never optimal to produce unless inventory empty)	—	2.6	0
$s = 5$	$0.2 \cdot 5 + 0.4 = 1.4$	—	—	1.4	0

$i=2$:

	$f_2(s,x_2) = hs + f_3^*(s - r_2 + x_2)$ $(+K$ if $s=0)$, $r_2=2$					
	$x_2=0$	$x_2=2$	$x_2=5$	$x_2=7$	f_2^*	x_2^*
$s=0$	—	$2.4+2=4.4$	$2.6+2=4.6$	$1.4+2=3.4$	3.4	7
$s=2$	$0.2\cdot 2+2.4=2.8$	—	—	—	2.8	0
$s=5$	$0.2\cdot 5+2.6=3.6$	—	—	—	3.6	0
$s=7$	$0.2\cdot 7+1.4=2.8$	—	—	—	2.8	0

$i=1$:

	$f_1(s,x_1) = hs + f_2^*(s - r_1 + x_1)$ $(+K$ if $s=0)$, $r_1=3$					
	$x_1=3$	$x_1=5$	$x_1=8$	$x_1=10$	f_1^*	x_1^*
$s=0$	$3.4+2=5.4$	$2.8+2=4.8$	$3.6+2=5.6$	$2.8+2=4.8$	4.8	5 or 10

There are two optimal schedules:

- produce all 10 jets in period 1
- produce 5 in period 1, 0 in period 2, 5 in period 3, 0 in period 4

Both lead to the minimal cost of 4.8 (million \$).