Stochastic Methods Lab, Jacobs University Bremen, Fall 2022 Session 3
Prof. Dr. Soeren Petrat Sep. 14, 2022

Recall: general ordinary annuity has present value

$$PV = C\,\frac{m}{r}\left(1-\left(1+\frac{r}{m}\right)^{-nm}\right), \quad m = \#\text{ of payments } C \text{ per year}, \ r = \text{annual interest rate}$$

Next:

Amortization:

Repay loan with regular payments (e.g., mortgages = loans for houses (or other real estate))

↳ payments for principal (repay loan) + interest

Traditional mortgage = equal regular payments C

What is C? Take formula for general ordinary annuity with PV = loan

$$\Rightarrow C = \underbrace{PV}_{=\text{loan}}\,\frac{r}{m}\left(1-\left(1+\frac{r}{m}\right)^{-nm}\right)^{-1}$$

The remaining principal after k payments (of amount C) is the (present) value after these payments:

$$\underset{\substack{\uparrow\\ \text{after } k \text{ payments}}}{PV_k} = C\sum_{i=1}^{nm-k}\left(1+\frac{r}{m}\right)^{-i}$$

= The value (after k payments) of the remaining nm-k cash-flows (this is what we would reasonably call "remaining value" or "remaining principal")

HW: create amortization schedule

Internal Rate of Return (IRR) / yield:

In a general cash-flow: given n and C_i, the present value as a function of r is

$$PV(r) := \sum_{i=1}^{n} \frac{C_i}{(1+r)^i}.$$

If we denote the real price P, then $\boxed{\text{the } r \text{ that solves } PV(r) = P \text{ is called IRR}}$.

Sometimes one defines the $\boxed{\text{net-present value } NPV(r) = PV(r) - P}$

Then IRR = zero of $NPV(r)$.