

Foundations of Mathematical PhysicsOrganization:

- Class: Wed/Fri, 9:45-11:00 (EH 8)
- Website: lecture notes, homework, news
- Grade: 100% final exam
 - ↳ bonus: 10% bonus from HW sheets (average HW score divided by 10)
 - ↳ note: bonus cannot change "fail" to "pass" grade
- TA: Gandeep Bhattarai
- Books:
 - lecture notes by Stefan Teufel / Marcello Porta (mostly based on Reed, Simon)
 - ~ first half of class
 - further references on website } for ~ second half of class

Topics:

Generally: Mathematics of (non-relativistic) quantum mechanics (QM)

Fields we touch upon:

- ↳ Math:
 - Analysis
 - Functional Analysis (Linear Algebra)
 - PDEs
- ↳ Physics:
 - Schrödinger eq.
 - Many-body QM
 - Bose-Einstein condensation (BEC)
 - Cold atoms, condensed matter physics

Structure of this class:

- Short introduction to quantum mechanics
- Fourier transform, distributions, free Schrödinger equation
- Hilbert space, self-adjoint operators, unitary groups, interacting Schrödinger eq.
(stop at bounded operators; briefly discuss results for unbounded operators)
- Non-linear Schrödinger equation, second quantization (Fock space), BEC, Bogoliubov theory

1. Introduction

1.1 Motivation

Classical physics: • Newton's eq.: $F = m \cdot a$

↳ initial positions and momenta determine trajectories of point particles

• with special (general) relativity: locality

↳ "nothing moves faster than the speed of light c "

Problems at beginning of 20th century: • blackbody radiation

• photoelectric effect

• stability of atoms/matter

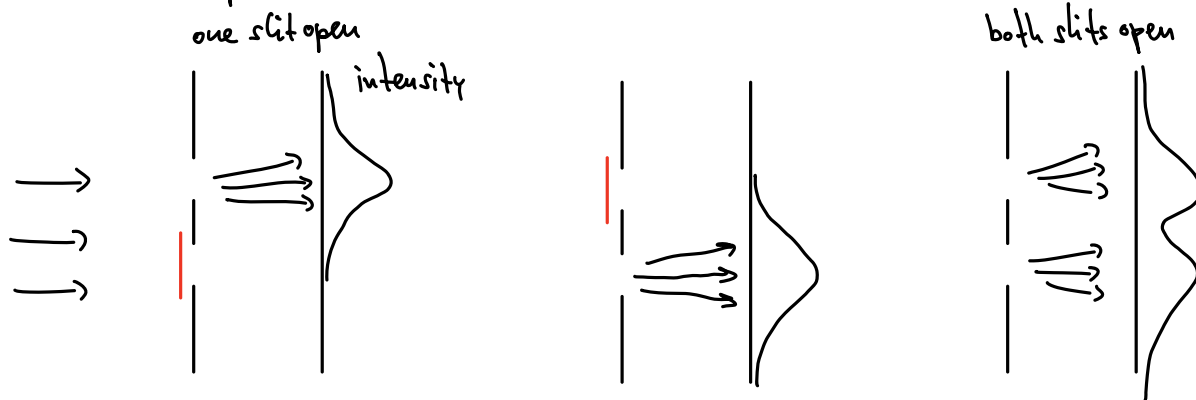
Solution to these problems led to Quantum Mechanics (QM)

↳ physics: Planck, Einstein, Schrödinger, Bohr, Heisenberg, Dirac, Born, de Broglie

↳ math: Hilbert, von Neumann

Example of "weird" quantum behavior: double slit experiment

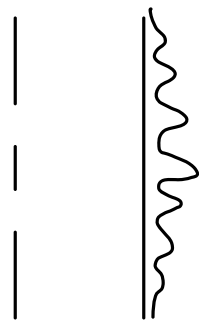
• classical particles:



- electromagnetic waves (light), water waves:

single slit similar

both slits open



interference pattern

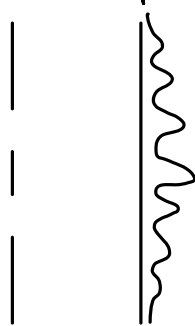
explanation:

+  = — destructive interference

+  =  constructive interference

- electrons:

both slits open

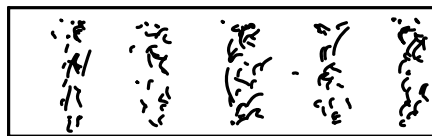


interference, big surprise!

maybe a collective wave?

single separated electrons:
(both slits open)

side view



single localized detection events \Rightarrow electrons are particles

interference pattern \Rightarrow electrons are waves and go through both slits at the same time

\Rightarrow seems like a paradox

Resolution of this paradox still hotly debated

Not debated: probabilistic description: some wave describes probability distribution of particle positions