

Operations Research

Homework 3

Due on September 27, 2023

Note: Your homework must be submitted via moodle (see the link on the class website) on the due day BEFORE THE TUTORIAL, i.e., before 20:45.

Problem 1 [7 points]

Suppose there is two foods:

- Food F_1 costs $c_1 = 12$ EUR per unit. It contains 60 calories, 12 micrograms of Vitamin A, and 10 milligrams of Vitamin C per unit.
- Food F_2 costs $c_2 = 15$ EUR per unit. It contains 65 calories, 6 micrograms of Vitamin A, and 30 milligrams of Vitamin C per unit.

The goal is to set up a diet that satisfies the following requirements with as little cost as possible:

- Consume at least 300 calories.
- Consume at least 36 micrograms of Vitamin A.
- Consume at least 90 milligrams of Vitamin C.

Formulate this problem as an LP problem (clearly writing down the objective function and all constraints). Then solve this LP problem graphically. More precisely: read off from a suitable picture where the optimal solution is, and then compute it exactly.

Problem 2 [5 points]

Write the following LP problem in standard form: *Maximize*

$$Z = 8x_1 + 12x_2 + 3x_3$$

subject to

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &\geq 8, \\ 4x_1 + 2x_2 + 4x_3 &\leq 9, \\ x_1 &\geq 0. \end{aligned}$$

Explicitly write down \mathbf{b} , \mathbf{c} and the decision variables \mathbf{x} as column vectors, and A as a matrix, with the notation used in class.

Problem 3 [8 points]

Let us revisit the WYNDOR company problem: *Maximize*

$$Z = 3x_1 + 5x_2$$

subject to

$$\begin{aligned}x_1 &\leq 4, \\2x_2 &\leq 12, \\3x_1 + 2x_2 &\leq 18, \\x_1, x_2 &\geq 0.\end{aligned}$$

- (a) Convert the problem to standard form.
- (b) Write down the corresponding simplex tableau (see Session 6).
- (c) Compute two basic feasible solutions.