Constructor University
Fall 2023

October 17, 2023

Operations Research

Homework 7

Due on October 25, 2023

Note: Your homework must be submitted via moodle (see the link on the class website) on the due day BEFORE THE TUTORIAL, i.e., before 20:45.

Problem 1 [6 points]

A construction company needs a heavy excavator for a three-year project. Since maintenance is expensive, it may be cheaper to replace the machine in the interim. The overall net cost (in 1000 €) of purchasing the excavator at the beginning of year i and trading it at the end of year j is as follows.

	Trade in end of year		
Purchase start of year	**1**	**2**	**3**
1	25	60	95
2	–	29	65
3	–	–	37

The task is to find the most economical schedule of purchasing and trading in the excavator(s).

(a) Draw a graphical representation and show that this is a shortest path problem.

(b) Solve the problem. This is most easily done by hand in this case.

Problem 2 [6 points]

(HL, Problem 9.4-3.) The Premiere Bank soon will be hooking up computer terminals at each of its branch offices to the computer at its main office using special phone lines with telecommunications devices. The phone line from a branch office need not be connected directly to the main office. It can be connected indirectly by being connected to another branch office that is connected (directly or indirectly) to the main office. The only requirement is that every branch office be connected by some route to the main office. The charge for the special phone lines is 100 $ times the number of miles involved, where the distance (in miles) between every pair of offices is as follows:

	Distance between Pairs of Offices					
	Main	**B.1**	**B.2**	**B.3**	**B.4**	**B.5**
Main Office	—	190	70	115	270	160
Branch 1	190	—	100	110	215	50
Branch 2	70	100	—	140	120	220
Branch 3	115	110	140	—	175	80
Branch 4	270	215	120	175	—	310
Branch 5	160	50	220	80	310	—

Management wishes to determine which pairs of offices should be directly connected by special phone lines in order to connect every branch office (directly or indirectly) to the main office at a minimum total cost.

(a) Describe how this problem fits the network description of the minimum spanning tree problem.

(b) Use the algorithm discussed in class to solve the problem.

Problem 3 [8 points]

(Based on HL, Problem 8.3-2.) Four cargo ships will be used for shipping goods from one port to four other ports (labeled 1, 2, 3, 4). Any ship can be used for making any one of these four trips. However, because of differences in the ships and cargoes, the total cost of loading, transporting, and unloading the goods for the different ship-port combinations varies considerably, as shown in the following table:

	Port			
	1	**2**	**3**	**4**
Ship 1	\$500	\$400	\$600	\$700
Ship 2	\$600	\$600	\$700	\$500
Ship 3	\$700	\$500	\$700	\$600
Ship 4	\$500	\$400	\$600	\$600

The objective is to assign the four ships to four different ports in such a way as to minimize the total cost for all four shipments.

(a) Let $x_{ij} = 1$ if ship i is sent to port j with $x_{ij} = 0$ otherwise. (A problem of this form is called an assignment problem.) Formulate and solve this problem in Pyomo. You should submit a your Ipython notebook, showing code and solution.

Hint: In Pyomo, you can declare a variable to take only values 0 and 1 by using the option `within=Boolean`.

(b) Now suppose you drop the requirement that the x_{ij} are Boolean and replace it with the usual non-negativity requirement. In other words, you are allowing to send "a fraction of a ship" to one port and another fraction to another. Re-solve the problem in Pyomo. What do you see?