Operations Research, Constructor University Bremen, Fall 2023 Session 1

Prof. Sören Petrat Sep. 5, 2023

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Class organization:

- website (all infos, lecture notes), MS Teams (tutorials, possibly some online sessions), moodle (HW submission)
- class: Tue 14:15-15:30, Thu 15:45-17:00, in-person
- tutorial: online, for now Wed: 20:45-22:00 (first tutorial: Wed Sep. 13)
- homework:
  - ↳ ~ weekly assignments (first HW already on website)
  - ↳ available on moodle and website
  - ↳ submission only via moodle (week after)
  - ↳ come to tutorial to get hints, ask questions, and get solution of previous sheet
- grade: • final exam only
  - • bonus: up to 5% from HW, up to 5% from midterm
    - ↳ HW: average of all but 2 worst HW sheets
  - Important: bonus cannot change fail grade to pass grade!
- TAs: Alexa Léon, Stasa Vasilic
  - ↳ weekly tutorials, question sessions
  - ↳ grading
- textbook: Hillier, Lieberman - Introduction to Operations Research (+ see website)

1. Introduction

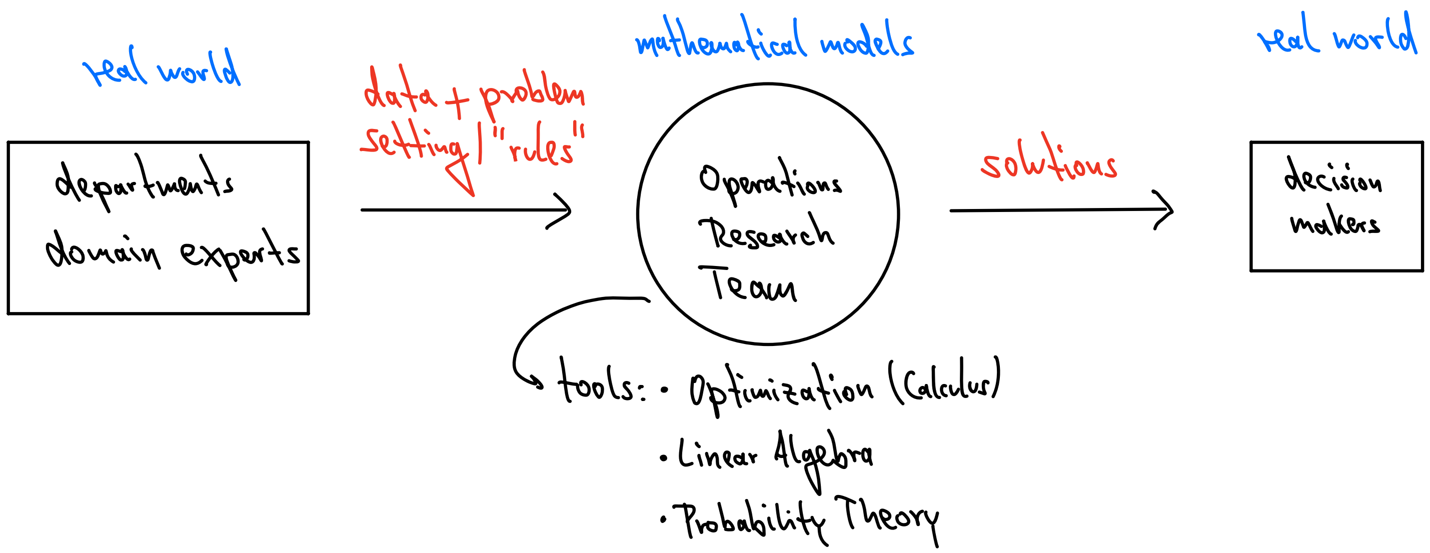
Operations Research (OR):

↳ scientific approach to management/planning problems for organizations (e.g., companies, government agencies, military)

↳ input: from many different departments; output: (near-) optimal solution(s)

↳ has led to immense savings (see textbook examples)

Schematically:



Key steps in OR problems:

1. Definition of the problem
2. Gather relevant data
3. Formulate a mathematical model
4. Solve the model (usually computer-based)

} We will focus on these steps

5. Test the model, sensitivity/error analysis → we will discuss this only briefly
6. Recommendation and/or implementation

Typical models / topics of this class:

- linear programming (better: linear optimization)
  - ↳ here: examples + theory ; computer implementation with pyomo library in Python
  - ↳ includes network optimization and transportation problems

$\sim \frac{1}{2}$ of this class

- Dynamic programming (can be linear or nonlinear)
- Decision theory (involves probability)
- Inventory theory
- Nonlinear programming

$\sim \frac{1}{2}$ of this class

Today, we start with a prototypical example (see also Hillier/Lieberman Ch. 3):

Wyndor Glass Co.

1. Problem setup:

↳ 3 plants:
- Plant 1: aluminum frames
- Plant 2: wood frames
- Plant 3: glass + final assembly

↳ 2 (new) products:
- Product 1: glass door with aluminum frame
- Product 2: wood-framed window

↳ Assume all that can be produced can be sold (marketing).

↳ Task: How many units of products 1 and 2 should be produced to maximize profit, subject to the available production capacities?

2. Data:

	required production time per batch (in hours)		available production time (in hours per week)
	Product 1	Product 2	
Plant 1	1	0	4
Plant 2	0	2	12
Plant 3	3	2	18
profit per batch	3000 \$	5000 \$	

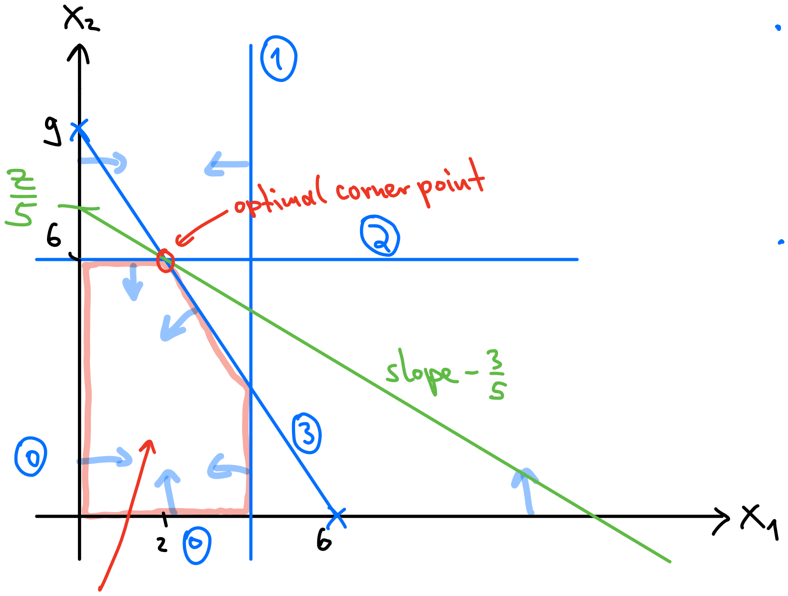
3. Mathematical model:

↳ Decision variables: • x_1 = # of batches of product 1 (per week)
• x_2 = # of batches of product 2 (per week)

↳ Objective function (here: profit): $Z = 3x_1 + 5x_2$ (in k \$), to be maximized

↳ Constraints:
• $x_1, x_2 \geq 0$ (0)
• $x_1 \leq 4$ (1)
• $2x_2 \leq 12$ (2) $(\Leftrightarrow x_2 \leq 6)$
• $3x_1 + 2x_2 \leq 18$ (3)

4. Graphical solution:



feasible region
= all potentially possible solutions

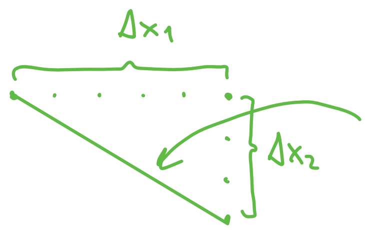
Constraint (3): Draw the boundary line $3x_1+2x_2 \le 18$

- It is the line that goes through points $(0,9)$ and $(6,0)$
  ($\uparrow x_1$, $\uparrow x_2$)
- Alternatively: solve for x_2:
  $x_2 = -\frac{3}{2}x_1 + 9$, i.e., it is a line with x_2-intercept $= 9$ and slope $-\frac{3}{2}$.
  (Note: Constraint is $x_2 \le -\frac{3}{2}x_1 + 9$, so the area under the line is feasible.)

Objective fct. $z = 3x_1 + 5x_2$

$\Rightarrow$ draw line $x_2 = -\frac{3}{5}x_1 + \frac{z}{5}$, i.e., line with slope $-\frac{3}{5}$. We want to maximize the intercept with x_2 axis $\frac{z}{5}$.

Recall:



slope $m = \frac{\Delta x_2}{\Delta x_1} = \frac{-3}{5}$

$\Rightarrow$ Optimal corner point: • read off: $x_1 = 2$, $x_2 = 6$

• or: (2): $2x_2 = 12 \Rightarrow x_2 = 6$

(3): $3x_1 + 2\underbrace{x_2}_{=6} = 18 \Rightarrow 3x_1 = 6 \Rightarrow x_1 = 2$

6. Recommendation:

↳ Produce 2 batches of product 1, and 6 batches of product 2

↳ Then profit will be maximal, namely $z = 3x_1 + 5x_2 = 3\cdot 2 + 5\cdot 6 = 36$, i.e., 36000 \$