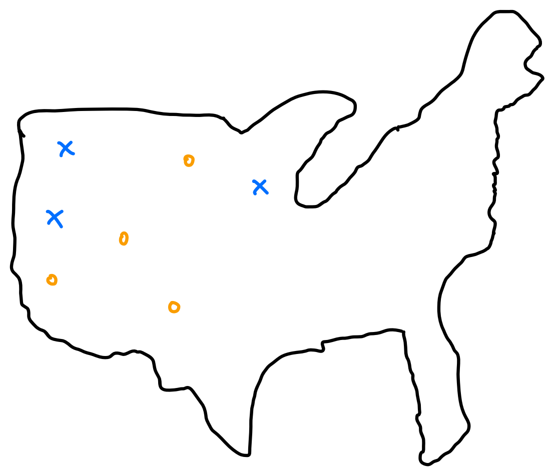
2.5 Transportation Problems

Example (Hillier, Lieberman Chapter 8): P & T company
↳ canned peas are prepared at canneries (x) in 3 cities across the US
↳ then shipped to 4 warehouses (o) across the US

Goal: minimize shipping cost but ensure supply



Data:

		Shipping cost per truckload				Output (supply)
		Warehouses				
		1	2	3	4	
Cannery	1	c_{11}	c_{12}	c_{13}	c_{14}	s_1
	2	c_{21}	c_{22}	c_{23}	c_{24}	s_2
	3	c_{31}	c_{32}	c_{33}	c_{34}	s_3
Allocation (demand)		d_1	d_2	d_3	d_4	

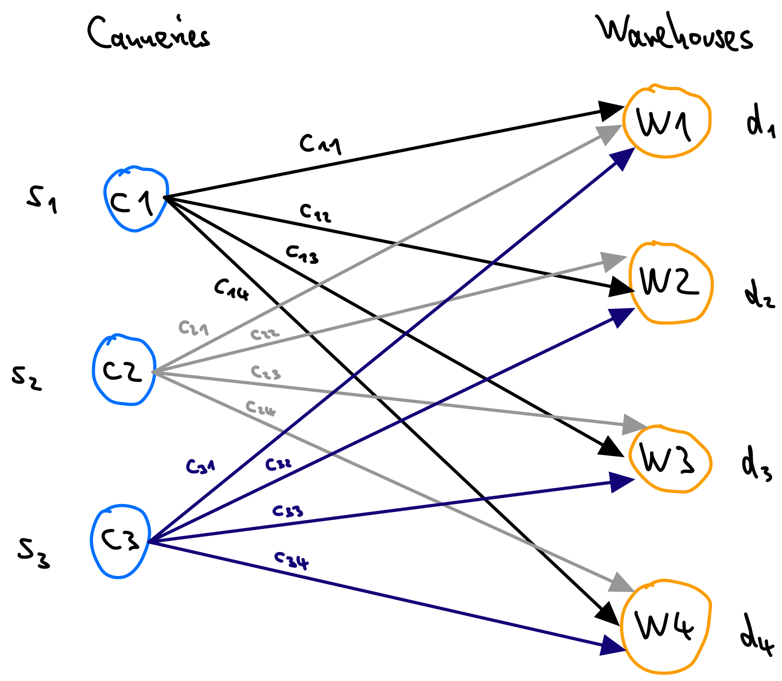
c_{ij}

s_i, $i = 1, \dots, m$ ($m = 3$ here)

d_j, $j = 1, \dots, n$ ($n = 4$ here)

decision variables: x_{ij} = number of truckloads shipped from cannery i to warehouse j

Network view of the transportation problem:



This leads to the following LP transportation problem:

- $\boxed{\text{Minimize}}$ transportation cost $Z = c_{11}x_{11} + c_{12}x_{12} + \dots + c_{mn}x_{mn} = \boxed{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}}$
- subject to: $x_{i1} + x_{i2} + x_{i3} + x_{i4} = s_i$ (everything is shipped away from cannery i)

  i.e., $\underbrace{\boxed{\sum_{j=1}^{n} x_{ij} = s_i}}$ for all $i = 1, \dots, m$ (all canneries)

  might want to relax this to $\sum_{j=1}^{n} x_{ij} \le s_i$ (at most as much as we have is shipped away)
  ↳ see next class

  and $x_{1j} + x_{2j} + x_{3j} = d_j$ (warehouse j receives the necessary supply)

  i.e., $\underbrace{\boxed{\sum_{i=1}^{m} x_{ij} = d_j}}$ for all $j = 1, \dots, n$ (all warehouses)

  might want to relax this to $\sum_{i=1}^{m} x_{ij} \ge d_j$ (warehouses receive at least the necessary supply)
  ↳ see next class

  and $\boxed{x_{ij} \ge 0.}$

Here, the constraints have a special pattern $(Ax = b)$:

$$\text{Matrix } A = \begin{array}{c} \begin{array}{cccccccccccc} x_{11} & x_{12} & x_{13} & x_{14} & x_{21} & x_{22} & x_{23} & x_{24} & x_{31} & x_{32} & x_{33} & x_{34} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & & & & & & & & \\ & & & & 1 & 1 & 1 & 1 & & & & \\ & & & & & & & & 1 & 1 & 1 & 1 \\ 1 & & & & 1 & & & & 1 & & & \\ & 1 & & & & 1 & & & & 1 & & \\ & & 1 & & & & 1 & & & & 1 & \\ & & & 1 & & & & 1 & & & & 1 \end{pmatrix} \end{array}, \quad b = \left(\begin{array}{l} \left.\begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix}\right\} \text{cannery constraints} \\ \left.\begin{matrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{matrix}\right\} \text{warehouse constraints} \end{array} \right)$$

For this type of problem the following holds:

- There are feasible solutions if and only if $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$ (supply = demand)

- If all s_i and d_j have integer values, then all basic variables in all basic feasible solutions have integer values. ← sometimes important for applications

- A streamlined simplex method is available. (We skip the details.)
  ↑ important for large scale problems

Next topics:
- What if supply ≠ demand?
- Other types of network optimization problems.
- A general framework for network optimization problems.