Now: consider additional difficulties:
- min. and max. demands
- Some connections are missing
- supply $\neq$ demand

We use the Metro Water District example (Hillier, Lieberman Ch. 8):
↳ Water from 3 rivers needs to be distributed to 4 cities (use all supply, minimize transportation costs)

	Transportation Costs				
	City 1	City 2	City 3	City 4	Supply
River 1	16	13	22	17	50
River 2	14	13	19	15	60
River 3	19	20	23	~~—~~ M	50
Minimum request	30	70	0	10	
Maximum request	50	70	30	~~∞~~ 60	

City 4 cannot be supplied with water from River 3.

We have upper and lower bounds for decision variables

Goal: Write this in the standard transportation problem form.

Note:

- Upper bound for City 4 can be replaced by $\underbrace{(50+60+50)}_{\text{total supply}} - \underbrace{(30+70)}_{\text{minimum needed by other cities}} = 60$
- We replace River 3 / City 4 entry by a very large cost M.
  ↳ then every optimal solution will have $x_{34} = 0$

- Problem: requested demand (210) ≥ supply (160)

We solve this by introducing a "dummy source" with a supply of 50 (= 210 − 160)

This leads to:

	Transportation Costs					
	City 1 (min.)	City 1 (extra)	City 2	City 3	City 4	Supply
River 1	16	16	13	22	17	50
River 2	14	14	13	19	15	60
River 3	19	19	20	23	M	50
Dummy	M	0	M	0	0	50
~~Minimum~~			~~70~~	~~0~~	~~10~~	
Demand	30	20	70	30	60	

50-30

actual demand shall not receive from dummy source

might receive from dummy source

already equality constraint

no minimum (if all $x_{ij} \geq 0$)

real supply 160, but maximum request by other cities is 50+70+30, so minimum 10 is always guaranteed

⇓

no need to introduce City 4 extra

The simplex method (pyomo) gives us the following result:

	1 min.	1 extra	2	3	4
1			50		
2			20		40
3	30	20			
4 (D)				30	20
	30	20	70	30	60

$z = 2460$

$\Rightarrow$ Actual water delivered:

- City 1: $30+20=\boxed{50}$
- City 2: $\boxed{70}$
- City 3: $30-30=\boxed{0}$
- City 4: $60-20=\boxed{40}$