Operations Research, Constructor University Bremen, Fall 2023 — Session 15

Prof. Sören Petrat — Oct. 26, 2023

Another example of network optimization: Project Management

Problem type 1:

- Given: set of activities taking time T_i to complete, and their dependencies (e.g., building construction)
- Goal: find minimal time to completion, and the corresponding order of activities (= critical path through network)
- Set up:
  - decision variables t_i = starting time of activity i
  - minimize t_{finish}
  - constraints: $t_j \geq t_i + T_i$ if j depends on i

    $t_{start} = 0$, $t_i \geq 0$

Problem type 2:

- Suppose a completion time is prescribed, but it is shorter than the critical path from above. Assume we can reduce the times of certain activities at a cost (this is called "crashing" an activity).
- Introduce x_i = units of time saved on activity i (decision variables)

  T_i = regular time for completion

  R_i = maximal time that can be saved

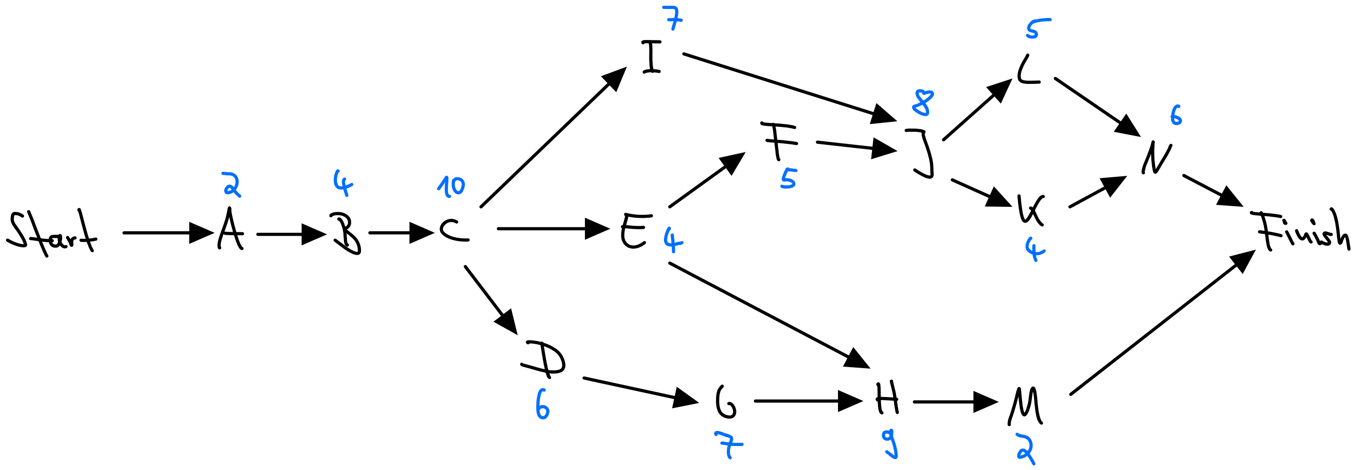
  c_i = cost of saving one unit of time
- LP problem: minimize cost $\sum_i c_i x_i$

  subject to $x_i \leq R_i$ for all activities i

  $t_j \geq t_i + (T_i - x_i)$

  $t_i \geq 0$, $x_i \geq 0$

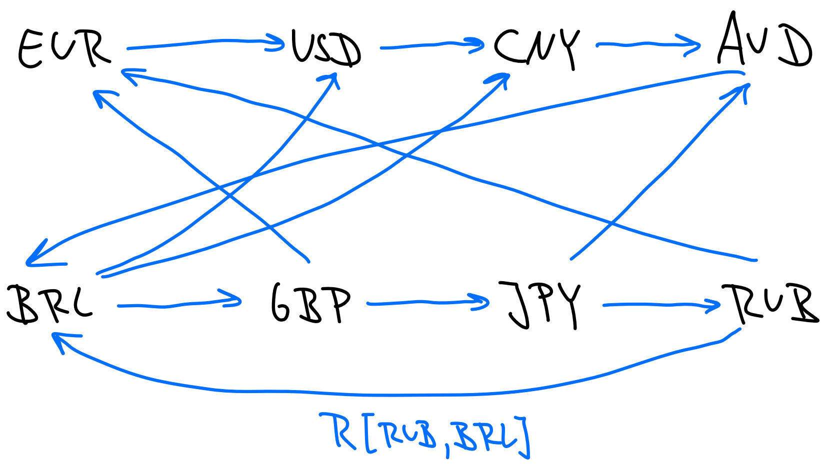
Example (Hillier, Liebermann Chapter 9.8 (9th edition)): Reliable Construction Company



Critical path = $\underbrace{\text{longest path}}_{\text{so all activities can be finished}}$ = A-B-C-E-F-J-L-N = 44 weeks

Suppose project needs to be completed in 40 weeks i.e., we need to crash some activities $\longrightarrow$ see promo code discussion.

Another example: Currency exchange rates and arbitrage (= risk-free profit)



Given: • list of currencies C

• list of exchange rates $R[\ldots,\ldots]$

• list of arcs A

Let us set up decision variables v_i = value of currency i, and a_{ij} = arbitrage for transaction $(i,j) \in A$.

Fix, e.g., $\underbrace{v[EUR] = 1}_{\text{constraint}}$, so all currency values are relative to EUR.

Now normally $v_i R_{ij} = v_j$ (value from exchanging currency i to j = value of currency j), but maybe there is arbitrage. So our constraints are $v_i R_{ij} = v_j + a_{ij}$.

True values of currencies are obtained from minimizing arbitrage $z = \sum_{(i,j) \in A} a_{ij}$.

See pyomo code for an example.

Some possible exam topics/questions:

- Formulate a given "text problem" as LP
- Solve LP problem graphically (also: shape of feasible region, number of solutions)
- Write LP problem in standard form
- Gaussian elimination and basic solutions
- Use simplex method to solve LP problem (what if feasible region is unbounded?)
- Shadow prices and their meaning
- Dual LP problems, weak and strong duality
- Transportation problems and their LP formulation
- Integer solution property, dummy variables
- Solve shortest path, minimum spanning tree, maximum flow problems
- Minimum cost flow problem and LP
- Pyomo: explain code; explain output; extract LP problem in mathematical notation from code; what happens if something is changed in the code

Good practice midterms: Fall 2021, Fall 2022 (see website)