We continue the example of the Hit and Miss Manufacturing Co.

- "Good" item is produced only with probability $p = \frac{1}{2}$
- For each new batch there are:
  - 300 \$ setup costs
  - 100 \$ cost per item
- At most 3 batches can be started, items can be inspected after each batch
- If no good item produced, there is a penalty of 1600 \$
- Objective: choose production schedule to minimize costs
- Decision variables: x_n = # of items to produce in batch / stage $n = 1,2,3$
- State s = # of acceptable items that still need to be produced = 0 or 1.

0: done, have produced a good one
1: no good item yet, might need to continue with next batch

$f_n(s_n, x_n)$ = expected cost for stages n onwards given state s_n, decision x_n, and optimal after

$$f_n^*(s_n) = \min_{x_n = 0,1,2,\dots} f_n(s_n, x_n)$$

Here: $f_n(0, x_n) = 0$ (no new batch is started if good item was already produced)

$$f_n(1, x_n) = \underbrace{K(x_n)}_{= \begin{cases} 0 & \text{for } x_n = 0 \\ 3 & \text{for } x_n > 0 \end{cases} = \text{setup costs}} + \underbrace{x_n}_{\text{cost per item}} + \underbrace{\left(\tfrac{1}{2}\right)^{x_n} f_{n+1}^*(1)}_{\text{expected costs if only bad items are produced; we start with } f_4^*(1) = 16}$$

all costs are in units of 100 \$

Solution:

Stage/batch $n=3$:

$$f_3(1,x_3) = K(x_3) + x_3 + \left(\tfrac{1}{2}\right)^{x_3} \cdot 16$$

s	$x_3=0$	$x_3=1$	$x_3=2$	$x_3=3$	$x_3=4$	$x_3=5$	...	$f_3^*(s)$	x_3^*
0	0	–	–	–	–	–	...	0	0
1	$0+0+16 = 16$	$3+1+8 = 12$	$3+2+4 = 9$	$3+3+2 = 8$	$3+4+1 = 8$	$3+5+\frac{1}{2} = 8\frac{1}{2}$	...(≥ 9)	8	3 or 4

$n=2$:

$$f_2(1,x_2) = K(x_2) + x_2 + \left(\tfrac{1}{2}\right)^{x_2} f_3^*(1)$$

s	$x_2=0$	$x_2=1$	$x_2=2$	$x_2=3$	$x_2=4$	...	f_2^*	x_2^*
0	0	–	–	–	–	...	0	0
1	$0+0+8 = 8$	$3+1+4 = 8$	$3+2+2 = 7$	$3+3+1 = 7$	$3+4+\frac{1}{2} = 7\frac{1}{2}$	...(≥ 8)	7	2 or 3

$n=1$:

$$f_1(1,x_1) = K(x_1) + x_1 + \left(\tfrac{1}{2}\right)^{x_1} f_2^*(1)$$

s	$x_1=0$	$x_1=1$	$x_1=2$	$x_1=3$	...	f_1^*	x_1^*
1	$0+0+7 = 7$	$3+1+\frac{7}{2} = 7\frac{1}{2}$	$3+2+\frac{7}{4} = 6\frac{3}{4}$	$3+3+\frac{7}{8} = 6\frac{7}{8}$	...(≥ 7)	$6\frac{3}{4}$	2

$\Rightarrow$ Optimal strategy: produce 2 items in first batch; if not successful,
2 or 3 items in second batch; if not successful,
3 or 4 items in third batch.

The associated minimal total expected cost is 675$.

3.2 Decision Analysis

We consider decisions to be made where consequences/outcomes are uncertain, e.g.,

- how much of a product sells (demand)
- supply availability
- whether or not to invest in equipment, securities, production facilities, ...

In practice, the following problem often arises:

- Prior probabilities are available for different scenarios, based on past experiences or intuition;
- We can invest in testing or experimentation to reduce uncertainties (= find better probabilities), e.g., test a product in a small market first, or get a more thorough analysis from experts/consultants.

  This costs extra, but often leads to higher expected profits.

Goal: maximize expected profit (or minimize expected costs, etc.)

Guiding example for this chapter: Goferbroke Oil Co. (Hillier, Lieberman: Ch. 15)

Setting:
- Company holds land where there might be oil or not (if not, land is "dry").
- Decision: Drill or sell?

	Payoff (in 1000$) in state	
Alternatives:	Oil	Dry
Drill (costs 100)	800-100=700	-100
Sell	90	90
prior probability	$\frac{1}{4}$	$\frac{3}{4}$

Given the prior probabilities, the expected payoffs are $\left(p = \frac{1}{4}\right)$:

- $\overbrace{\mathbb{E}[\text{Drill}]}^{\text{expected payoff if we drill}} = 700p - 100(1-p) = \frac{700}{4} - \frac{300}{4} = 100$
- $\mathbb{E}[\text{Sell}] = 90p + 90(1-p) = 90$

$\Rightarrow$ Drilling seems preferable here.

Note: For what probability p is it worth drilling?

Want $700p - 100(1-p) \geq 90 \Rightarrow 800p \geq 190 \Rightarrow p \geq \frac{190}{800} \approx 0.24$

This is very close to $\frac{1}{4}$, so some experimentation is advisable.

For this example: We can do a seismic survey to find better probabilities:

- cost: 30 (k$)
- probabilities:

$$\left.\begin{array}{ll} \overset{\mathbb{P}(F|\text{Oil})}{\mathbb{P}(\text{Favorable}|\text{Oil})} = 0.6, & \overset{\mathbb{P}(\text{Unf}|\text{Oil})}{\mathbb{P}(\text{Unfavorable}|\text{Oil})} = 0.4 \\ \underbrace{\mathbb{P}(\text{Favorable}|\text{Dry})}_{\text{probability for favorable outcome if land is dry}} = 0.2, & \mathbb{P}(\text{Unfavorable}|\text{Dry}) = 0.8 \end{array}\right\} \text{conditional probabilities}$$