

Recall our example from last time:

Alternatives:	Payoff (in 1000\$) in state	
	Oil	Dry
Drill (costs 100)	$800 - 100 = 700$	-100
Sell	90	90
prior probability	$\frac{1}{4}$	$\frac{3}{4}$

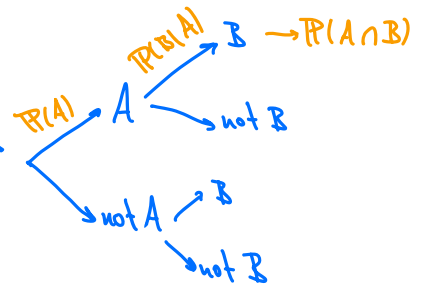
For this example: We can do a seismic survey to find better probabilities:

- Cost: 30 (k\$)
- Probabilities: (Recall:  $TP(A|B)$  = probability for A given B; also called conditional probability.)
  - Suppose there is oil. Then the probability of the experimentation to report
    - favorable conditions is  $TP(\text{Favorable} | \text{Oil}) = TP(F | \text{Oil}) = 0.6$
    - unfavorable conditions is  $TP(\text{Unfavorable} | \text{Oil}) = TP(\text{Unf} | \text{Oil}) = 0.4$  ( $\rightarrow$  false negative)
  - Suppose there is no oil. Then the prob. of exp. to report
    - favorable conditions is  $TP(\text{Favorable} | \text{Dry}) = TP(F | \text{Dry}) = 0.2$
    - unfavorable conditions is  $TP(\text{Unfavorable} | \text{Dry}) = TP(\text{Unf} | \text{Dry}) = 0.8$

But we base our decision on the report outcome (favorable or unfavorable). So now we should find probabilities for oil given favorable/unfavorable outcome, i.e.,  $TP(\text{Oil} | F)$ ,  $TP(\text{Oil} | \text{Unf})$ .

Recall:  $\underbrace{P(A \cap B)}_{\text{prob. for } A \text{ and } B} = \underbrace{P(A|B)}_{\text{prob. for } A \text{ given } B} \underbrace{P(B)}_{\text{prob. for } B} = P(B \cap A) = P(B|A)P(A)$

$$\Rightarrow P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (\text{Bayes' rule})$$



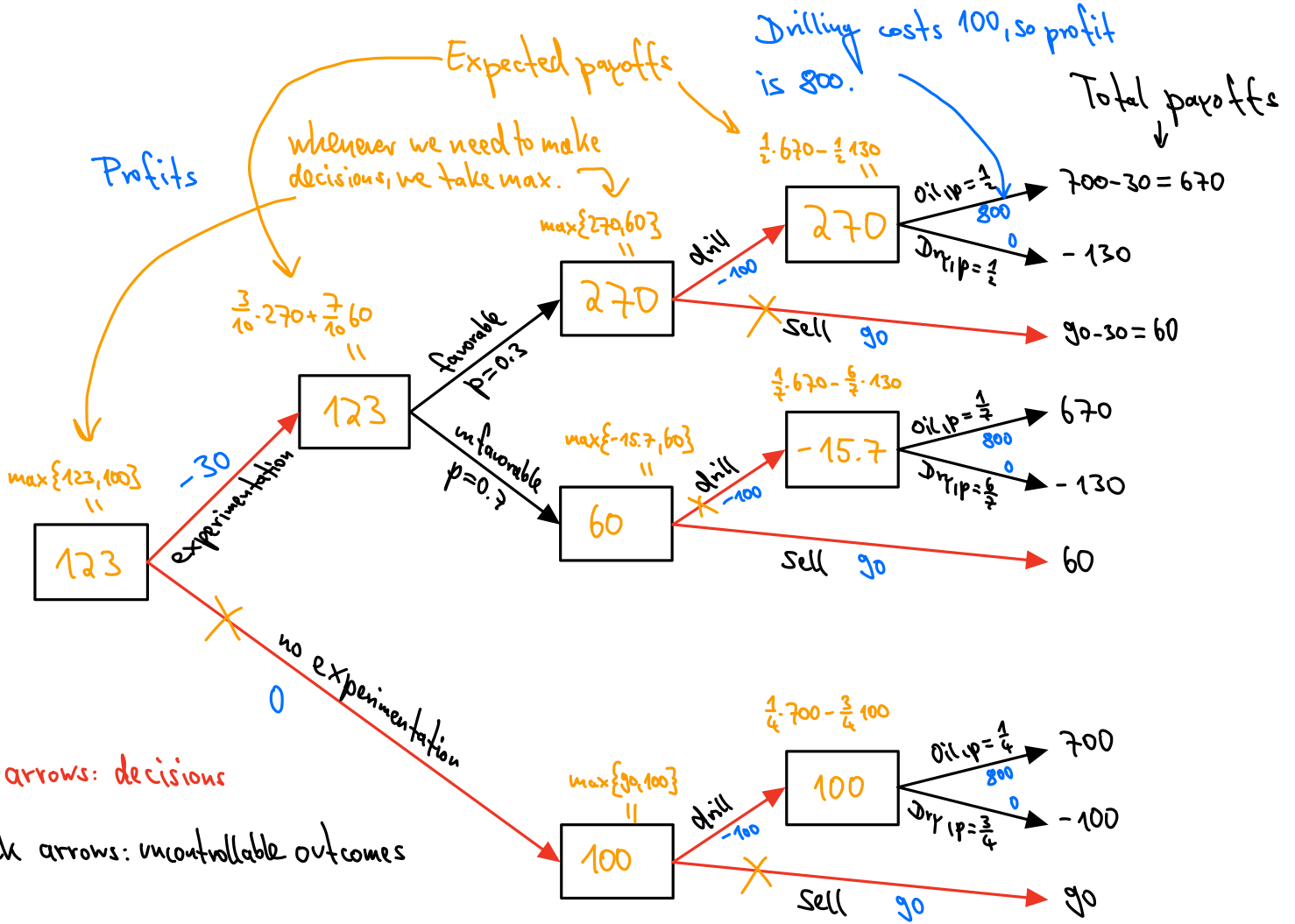
Let us first compute  $P(F)$ :

$$\begin{aligned} P(F) &= P(F|oil)P(oil) + P(F|Dng)P(Dng) \\ &= 0.6 \cdot \frac{1}{4} + 0.2 \cdot \frac{3}{4} \\ &= \frac{6}{40} + \frac{6}{40} = \frac{12}{40} = \frac{3}{10} = 0.3 \quad \Rightarrow P(Vnf) = 0.7 \end{aligned}$$

$$\Rightarrow P(oil|F) = \frac{P(F|oil) \cdot P(oil)}{P(F)} = \frac{0.6 \cdot \frac{1}{4}}{0.3} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$P(oil|Vnf) = \frac{0.4 \cdot \frac{1}{4}}{0.7} = \frac{1}{7}$$

Our computations lead us to the following decision tree:



red arrows: decisions

black arrows: uncontrollable outcomes

Result: • Do experimentation

• If favorable: drill

• If unfavorable: sell

• The overall expected profit is 123 000 \$.