

Next, we consider **periodic-review models**.

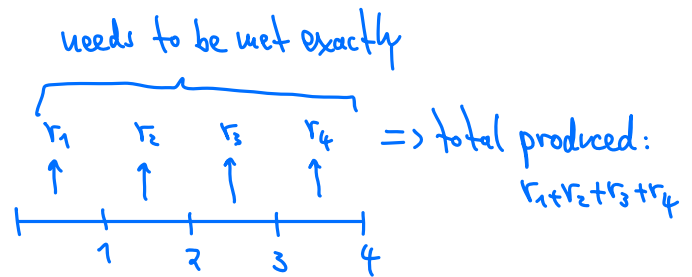
Setup: • n periods

• r_i = demand in period i

• as before: • K = setup cost (for producing/purchasing)

• c = unit cost of producing/purchasing

• h = holding cost for each unit left in inventory at end of period



Goal: find optimal production schedule (to minimize cost)

E.g.: • produce all at beginning: low setup costs, but high holding costs

• produce only when necessary: high setup costs, no holding costs

Note: Every production schedule satisfies total demand $\sum_{i=1}^n r_i$, so the unit cost c is irrelevant for finding the optimal schedule. We will drop it from now on.

Example: Small airplane manufacturer (Hillier, Lieberman: Chapter 18.4)

• customer orders 10 jets

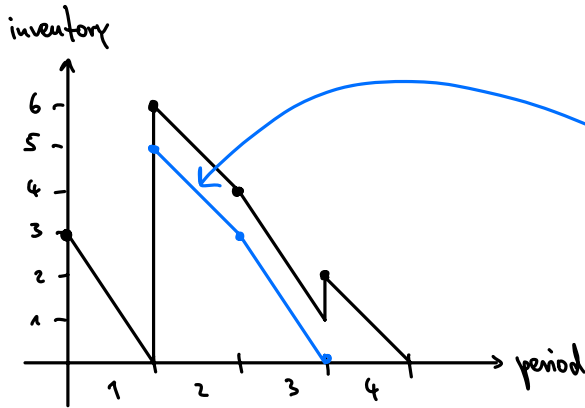
• delivery in 4 periods: $r_1 = 3$, $r_2 = 2$, $r_3 = 3$, $r_4 = 2$

• setup costs: 2 (million \$)

• holding costs: 0.2 (million \$) per jet per period

Note:

An example of a production schedule is:



produce: 3 6 0 1 \Rightarrow total = 10
 deliver: 3 2 3 2 \Rightarrow total = 10
 cost: K K+4h h K \Rightarrow total cost: 3K+5h

But this cannot be optimal. We should only produce when inventory is empty (otherwise we can find a schedule with same setup costs, but lower holding cost).

This problem setup begs for a dynamic programming treatment:

- state s = # of jets in inventory
- x_i = jets produced in period i

$$f_i(s, x_i) = \begin{cases} hs + f_{i+1}^*(s - r_i + x_i) + K, & s=0 \quad \leftarrow \text{produce (only when inventory empty, i.e., } s=0) \\ hs + f_{i+1}^*(s - r_i + x_i), & s>0 \quad \leftarrow \text{don't produce} \end{cases}$$

inventory - demand + produced = # units in next inventory

Solution:

$$i=4: f_4^*(s) = \begin{cases} K=2 & \text{if } s=0 \\ hs = 0.2 \cdot 2 = 0.4 & \text{if } s=2 \end{cases}$$

$i=3:$

	$x_3=0$	$x_3=3$	$x_3=5$	f_3^*	x_3^*
$s=0$	(need to satisfy demand)	$2+2=4$	$0.4+2=2.4$	2.4	5
$s=3$	$0.2 \cdot 3 + 2 = 2.6$	(never optimal to produce unless inventory empty)	—	2.6	0
$s=5$	$0.2 \cdot 5 + 0.4 = 1.4$	—	—	1.4	0

$i=2:$

$$f_2(s, x_2) = hs + f_3^*(s - x_2 + x_2) \quad (\text{+K if } s=0) \quad \text{with } r_2=2$$

	$x_2=0$	$x_2=2$	$x_2=5$	$x_2=7$	f_2^*	x_2^*
$s=0$	—	$2.4+2=4.4$	$2.6+2=4.6$	$1.4+2=3.4$	3.4	7
$s=2$	$0.2 \cdot 2 + 2.4 = 2.8$	—	—	—	2.8	0
$s=5$	$0.2 \cdot 5 + 2.6 = 3.6$	—	—	—	3.6	0
$s=7$	$0.2 \cdot 7 + 1.4 = 2.8$	—	—	—	2.8	0

$i=1:$

$$f_1(s, x_1) = hs + f_2^*(s - x_1 + x_1) \quad (\text{+K if } s=0) \quad \text{with } r_1=3$$

	$x_1=3$	$x_1=5$	$x_1=8$	$x_1=10$	f_1^*	x_1^*
$s=0$	$3.4+2=5.4$	$2.8+2=4.8$	$3.6+2=5.6$	$2.8+2=4.8$	4.8	5 or 10

There are two optimal schedules:

- produce all 10 jets in period 1
- produce 5 in period 1, 0 in period 2, 5 in period 3, 0 in period 4

Both lead to the minimal cost of 4.8 (million \$).