

Elements of Linear Algebra

Homework 1 (covering Weeks 1 and 2)

Due on September 18, 2024, before the tutorial! Please submit on moodle.

Problem 1 [5 points]

Prove the following identities for vectors $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$.

1. The “*BAC-CAB-identity*”

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b}).$$

2. The *Jacobi identity* in three dimensions

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}.$$

Problem 2 [5 points]

Prove the following identities for vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathbb{R}^3$.

1. The *Cauchy-Binet formula* in three dimensions

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}).$$

Hint: Use the identity $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{v} \cdot (\mathbf{w} \times \mathbf{u})$.

2. The identity

$$\|\mathbf{a} \times \mathbf{b}\|^2 = \|\mathbf{a}\|^2 \|\mathbf{b}\|^2 - (\mathbf{a} \cdot \mathbf{b})^2.$$

Problem 3 [5 points]

Find an equation for the plane that contains the point $\mathbf{p} = (2, 4, 6)$ and the line

$$\mathbf{x} = \begin{pmatrix} 7 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}.$$

Problem 4 [5 points]

Find the distance between the point $\mathbf{p} = (1, 2, 3)$ and the line

$$\mathbf{x} = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$