Constructor University
Fall 2024

September 16, 2024

Elements of Linear Algebra

Homework 2 (covering Weeks 3 and 4)

Due on September 30, 2024, before the tutorial! Please submit on moodle.

Problem 1 [5 points]

Prove the following statement: Let $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ be linearly independent. If a vector $\boldsymbol{w}$ can be written

$$\boldsymbol{w} = \sum_{k=1}^{n} \alpha_k \, \boldsymbol{v}_k \,,$$

the choice of the coefficients $\alpha_1, \ldots, \alpha_n$ is unique.

Problem 2 [5 points]

Solve the following system of linear equations using the method taught in class.

$$\begin{aligned} x_1 + 3\,x_2 - 5\,x_3 &= 4 \\ x_1 + 4\,x_2 - 8\,x_3 &= 7 \\ -3\,x_1 - 7\,x_2 + 9\,x_3 &= -6 \end{aligned}$$

Problem 3 [5 points]

Find conditions on α such that following system of linear equations has (a) exactly one solution, (b) no solutions, or (c) an infinite number of solutions; give all solutions where they exist.

$$\begin{aligned} x_1 + \alpha\,x_2 &= 1 \\ x_1 - x_2 + 3\,x_3 &= -1 \\ 2\,x_1 - 2\,x_2 + \alpha\,x_3 &= -2 \end{aligned}$$

Problem 4 [5 points]

Solve the following system of linear equations using the method taught in class.

$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 2 \\ 2x_1 + 6x_2 \qquad - x_4 &= 1. \end{aligned}$$