

Elements of Linear Algebra

Homework 6 (covering Weeks 11, 12, 13)

Due on December 2, 2024, before the tutorial! Please submit on moodle.

Problem 1 [16 points]

Consider the matrix

$$A = \begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}.$$

Compute

- (a) if possible, the diagonalization,
- (b) the LU decomposition with the convention that L has 1's on the diagonal,
- (c) if possible, a Cholesky decomposition,
- (d) a QR decomposition in two ways: once with Gram-Schmidt, once using Householder reflections,
- (e) a singular value decomposition.

Then, use each of these decompositions to compute A^{-1} and $\det(A)$. Use the LU , QR , and singular value decompositions to find the solutions to $Ax = b$ with $b = (1, 2)^T$.

(Note: Here, we are just considering a simple 2×2 matrix example to illustrate all these methods. Of course, inverses, determinants and solutions to linear equations can be easily found in other ways here.)

Problem 2 [4 points]

Consider the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and the following slight perturbation of this matrix,

$$B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ \frac{1}{60000} & 0 & 0 & 0 \end{pmatrix}.$$

Compute

- (a) the eigenvalues and eigenvectors of A and B ,
- (b) a singular value decomposition of A and B .

What can you conclude?