Week 1: Basic Calculus Review

1. MULTI Single

Find the (complex) roots of the polynomial

$$p(x) = x^2 + 4\,x + 13$$

(a) $x_1 = +2 + 3i$, $x_2 = +2 - 3i$
(b) $x_1 = -3 + 2i$, $x_2 = -3 - 2i$
(c) $x_1 = +3 - 2i$, $x_2 = +3 + 2i$
(d) $x_1 = -2 - 3i$, $x_2 = -2 + 3i$

2. MULTI Single

Let $p(x)$ be a polynomial of degree n with **arbitrary complex coefficients**. Which of the following is true?

(a) If $p(x) = c(x - \alpha_1)(x - \alpha_2)...(x - \alpha_n)$ with $\alpha_1, ..., \alpha_n \in \mathbb{R}$, then the roots of $p(x)$ can be real and also imaginary.
(b) $p(x)$ has exactly n roots (considering multiplicities)
(c) $p(x)$ can have no roots
(d) If z is a root, then its complex conjugate $\overline{z}$ is also a root

3. MULTI Single

Find all the values of the parameter λ for which the equation

$$2\,x^2 - \lambda\,x + \lambda = 0$$

has no real solutions.

(a) $\lambda \in \{0, 8\}$
(b) $\lambda \in (0, 8)$
(c) $\lambda \in (-\infty, 0) \cup (8, \infty)$
(d) $\lambda \in (-8, 0)$

4. MULTI Multiple

The number $5.21\overline{37}$ is:

(a) an integer
(b) a real number
(c) a rational number
(d) a natural number

5. MULTI Single

Assuming that $z = a + bi$ is a complex number, compute real and imaginary part of $\dfrac{1}{z^2}$

(a) $\mathrm{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 - b^2}{(a^2 - b^2)^2}, \mathrm{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{2ab}{(a^2 - b^2)^2}$
(b) $\mathrm{Re}\left(\dfrac{1}{z^2}\right) = \dfrac{a^2 - b^2}{(a^2 + b^2)^2}, \mathrm{Im}\left(\dfrac{1}{z^2}\right) = \dfrac{-2ab}{(a^2 + b^2)^2}$

(c) $\operatorname{Re}\left(\frac{1}{z^2}\right) = \frac{a^2+b^2}{(a^2+b^2)^2}, \operatorname{Im}\left(\frac{1}{z^2}\right) = \frac{-2ab}{(a^2+b^2)^2}$
(d) $\operatorname{Re}\left(\frac{1}{z^2}\right) = \frac{a^2+b^2}{(a^2+b^2)^2}, \operatorname{Im}\left(\frac{1}{z^2}\right) = \frac{2ab}{(a^2+b^2)^2}$

6. MULTI Single

Let $p(x)$ be a polynomial of degree n with **real** coefficients. Which of the following is true?

(a) $p(x)$ can have less than n complex roots
(b) If $p(x)$ is odd, it can have no roots
(c) If z is a root, then its complex conjugate is z^* is also a root
(d) $p(x)$ has n distinct real roots

7. MULTI Single

Compute $\left|\frac{1+i}{2-i}\right|$.

(a) $\left|\frac{1+i}{2-i}\right| = \frac{2}{3}$
(b) $\left|\frac{1+i}{2-i}\right| = \frac{2}{5}$
(c) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{3}}$
(d) $\left|\frac{1+i}{2-i}\right| = \sqrt{\frac{2}{5}}$

8. MULTI Single

Which of the following does not describe the rational numbers $\mathbb{Q}$?

(a) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{Z} \text{ and } m \neq 0\right\}$
(b) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{N}\right\} \cup \left\{\frac{-n}{m} \mid n, m \in \mathbb{N}\right\} \cup \{0\}$
(c) $\mathbb{Q} = \left\{\frac{n}{m} \mid n, m \in \mathbb{N}\right\}$
(d) $\mathbb{Q} = \left\{\frac{n}{m} \mid n \in \mathbb{Z} \text{ and } m \in \mathbb{N}\right\}$

9. MULTI Single

Let $g(x) = x^2 + 1$. Determine the domain and range of $g(x)$.

(a) $Domain(g) = [0, \infty)$, $Range(g) = (-\infty, \infty)$
(b) $Domain(g) = (-\infty, \infty)$, $Range(g) = [0, \infty)$
(c) $Domain(g) = (-\infty, \infty)$, $Range(g) = [1, \infty)$
(d) $Domain(g) = [0, \infty)$ $Range(g) = (-\infty, \infty)$

10. MULTI Single

Let $f(x) = 2^{-9x+3}$. Determine the domain and range of $f(x)$ and its inverse $f^{-1}(x)$.

(a) $Dom(f) = (-\infty, \infty)$, $Ran(f) = (0, \infty)$,
$Dom(f^{-1}) = (0, \infty)$, $Ran(f^{-1}) = (-\infty, \infty)$

(b) $Dom(f) = [0, \infty)$, $Ran(f) = [0, \infty)$,
$Dom(f^{-1}) = [0, \infty)$, $Ran(f^{-1}) = [0, \infty)$

(c) $Dom(f) = (0, \infty)$, $Range(f) = (-\infty, \infty)$,
$Dom(f^{-1}) = (-\infty, \infty)$, $Ran(f^{-1}) = (0, \infty)$

(d) $Dom(f) = (-\infty, \infty)$, $Ran(f) = [0, \infty)$,
$Dom(f^{-1}) = [0, \infty)$, $Ran(f^{-1}) = (-\infty, \infty)$

Total of marks: 10