Week 2: Elementary Analytical Geometry

1. MULTI Single

   What is the angle (in radian, i.e., where $360°$ corresponds to 2π) between the vectors

   $$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

   and

   $$\begin{bmatrix} 3 \\ 7 \\ 17 \end{bmatrix}?$$

   (a) $\frac{\pi}{2}$
   (b) $\frac{\pi}{4}$
   (c) π
   (d) 0

2. MULTI Single

   If $\vec{u}$ and $\vec{v}$ are perpendicular unit vectors, then

   (a) $|\vec{u} - \vec{v}| = 1$
   (b) $|\vec{u} - \vec{v}| = \sqrt{2}$
   (c) $|\vec{u} - \vec{v}| = 0$
   (d) $|\vec{u} - \vec{v}|$ cannot be computed without further information on $\vec{u}$ and $\vec{v}$

3. MULTI Single

   How long is the vector $(1, 1, \ldots, 1)$ in 16 dimensions?

   (a) Length $= 1$
   (b) Length $= 4$
   (c) Length $= 16$
   (d) Length $= 32$

4. MULTI Single

   Which of the following formulas is not true (for $\vec{u}, \vec{v} \in \mathbb{R}^n$)?

   (a) $|\vec{u} \times \vec{v}| \leq |\vec{u}||\vec{v}|$
   (b) $\cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2|\vec{v}|^2}$
   (c) $|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 - 2|\vec{u}||\vec{v}|\cos(\theta) + |\vec{v}|^2$
   (d) $|\vec{u} + \vec{v}| \leq |\vec{u}| + |\vec{v}|$

5. MULTI Single

   Let x, y, z be such that $x + y + z = 0$. Define $u = (x, y, z)$ and $v = (z, x, y)$. What is the value of $\frac{u \cdot v}{|u||v|}$?

   (a) $\frac{-x^2 + yz}{x^2 + y^2 + (x - y)^2}$

(b) $-\dfrac{1}{2}$
(c) 1
(d) 0

6. MULTI Single

A line is given by $\vec{r} = \lambda\vec{a} + \vec{b}$, with $\vec{a} = (1, -1, 4)$ and $\vec{b} = (4, 5, 6)$, while the equation of a plane is given by $-2x + 2y + z = 17$. What are the coordinates of the point P where the line and plane intersect?

(a) The line and the plane intersect infinitely many times
(b) $P = (3, 3, 17)$
(c) The line and the plane do not intersect
(d) $P = (-1, 4, 7)$

7. MULTI Single

What is the equation of the hyperplane, given by $\begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} = \vec{p_0} + \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$ with

$$\vec{p_0} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \vec{c} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \alpha, \beta, \gamma \in \mathbb{R}$$

(a) $t + x - y - z - 1 = 0$
(b) $t + x - y + z - 1 = 0$
(c) $-t - x - y - z + 1 = 0$
(d) $-t - x - y + z + 1 = 0$

8. MULTI Single

Find the cross product $\vec{u} \times \vec{v}$ of $\vec{u} = \langle 3, 2, -1\rangle$, $\vec{v} = \langle 1, 1, 0\rangle$

(a) $\langle -1, -1, 5\rangle$
(b) $\langle -6, -4, 2\rangle$
(c) $\langle 1, -1, 1\rangle$
(d) $\langle 6, -4, 2\rangle$

9. MULTI Single

Find the unit vector along the direction of the cross product $\vec{u}\times\vec{v}$ of $\vec{u} = \langle 7, -1, 3\rangle$, $\vec{v} = \langle 2, 0, -2\rangle$.

(a) $\dfrac{1}{\sqrt{108}}\langle -2, -10, 2\rangle$
(b) $\dfrac{1}{408}\langle 2, 20, 2\rangle$
(c) $\dfrac{1}{\sqrt{408}}\langle 2, 20, 2\rangle$
(d) $\dfrac{1}{108}\langle -2, -10, 2\rangle$

10. MULTI Single

Let $\epsilon_{ijk} = \begin{cases} 1 & \text{if } (i\,j\,k) = (1\,2\,3),\ (2\,3\,1),\ \text{or } (3\,1\,2) \\ -1 & \text{if } (i\,j\,k) = (1\,3\,2),\ (3\,2\,1),\ \text{or } (2\,1\,3) \\ 0 & \text{else} \end{cases}$

Consider $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$. Which of the following is equivalent to the kth component of $\vec{u} \times \vec{v}$

(a) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk} u_i\, v_j$

(b) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk} (u_i\, v_j + v_i\, u_j)$

(c) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk} (u_i\, v_j - v_i\, u_j)$

(d) $[\vec{u} \times \vec{v}]_k = \sum_{i,j=1}^{3} \epsilon_{ijk} v_i\, u_j$

Total of marks: 10