Week 7: The determinant and eigenvalues

1. MULTI Single

Consider the linear transformation given by the matrix.

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

Is T invertible? Why? (Hint: Pay attention to the first three rows.)

(a) T is invertible because $\det(T) = 0$
(b) T is not invertible because $\det(T) \neq 0$
(c) T is invertible because $\det(T) \neq 0$
(d) T is not invertible because $\det(T) = 0$

2. MULTI Single

Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ that has the following values on the basis elements:

$$T : (1,0,0) \mapsto (-1,0,1)$$
$$T : (0,1,0) \mapsto (3,-2,-1)$$
$$T : (0,0,1) \mapsto (1,1,1)$$

Is T invertible? Why?

(a) T is invertible because $\det(T) = 0$
(b) T is not invertible because $\det(T) = 0$
(c) T is invertible because $\det(T) \neq 0$
(d) T is not invertible because $\det(T) \neq 0$

3. MULTI Single

Use Cramer's rule to solve for y in

$$A\vec{x} = \begin{bmatrix} 2 & 6 & 2 \\ 1 & 4 & 2 \\ 5 & 9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \vec{b}.$$

(a) 2
(b) -1
(c) 1
(d) -2

4. MULTI Single

Use Cramer's rule to solve for y in

$$ax + by + cz = 1$$
$$dx + ey + fz = 0$$
$$gx + hy + iz = 0.$$

You can assume that the relevant 3×3 matrix has non-zero determinant D.

(a) $\dfrac{ei - fh}{D}$

(b) $\dfrac{fh - ei}{D}$

(c) $\dfrac{fg - di}{D}$

(d) $\dfrac{di - fg}{D}$

5. MULTI Single

Use Cramer's rule to solve for x_1 in

$$\begin{aligned} 2x_1 + x_2 &= 1 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 0. \end{aligned}$$

(a) $\dfrac{1}{2}$

(b) $\dfrac{3}{4}$

(c) 3

(d) $\dfrac{1}{4}$

6. MULTI Single

Compute the inverse of the matrix (try using the classical adjoint)

$$H = \begin{bmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

where a, b, c are arbitrary real numbers.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ab - c & -b & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ c - ab & b & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -a & ab - c \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & a & c - ab \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$

7. MULTI Single

Compute the inverse of the matrix (try using the classical adjoint)

$$R_\theta = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

where θ is an arbitrary real number.

(a) $R_{\theta^{-1}}$
(b) $-R_\theta$
(c) $-R_\theta$
(d) $R_{-\theta}$

8. MULTI Single

Find the eigenvalues and the associated eigenvectors of the matrix

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

(a) $\lambda_1 = \sqrt{2}, v_1 = (1, \sqrt{2}-1)$
$\lambda_2 = -\sqrt{2}, v_2 = (1, -\sqrt{2}-1)$
(b) $\lambda_1 = 1, v_1 = (\sqrt{2}-1, 1)$
$\lambda_2 = -1, v_2 = (\sqrt{2}+1, 1)$
(c) $\lambda_1 = 1, v_1 = (1, \sqrt{2}-1)$
$\lambda_2 = -1, v_2 = (1, -\sqrt{2}-1)$
(d) $\lambda_1 = \sqrt{2}, v_1 = (1, 1)$
$\lambda_2 = -\sqrt{2}, v_1 = (1, -1)$

9. MULTI Single

Find the eigenvalues and the associated eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

(a) $\lambda_1 = 0, v_1 = (-2, -1)$
$\lambda_2 = 5, v_2 = (-1, 2)$
(b) $\lambda_1 = 0, v_1 = (2, 1)$
$\lambda_2 = 3, v_2 = (1, 3)$
(c) $\lambda_1 = 0, v_1 = (2, -1)$
$\lambda_2 = 5, v_2 = (1, 2)$
(d) $\lambda_1 = 1, v_1 = (2, 1)$
$\lambda_2 = 5, v_2 = (1, 2)$

10. MULTI Single

Find all eigenvalues and associated eigenvectors of the matrix

$$A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}.$$

(a) $\lambda_1 = \lambda_2 = -2, v = (t, s, 3t)$ for $s, t \in \mathbb{R}$ and $\lambda_3 = 1, v_3 = (1, -1, 2)$
(b) All eigenvalues are zero.
(c) $\lambda_1 = \lambda_2 = 1, v = (1, 0, 3)$ and $\lambda_3 = 2, v_3 = (1, -1, 2)$
(d) $\lambda_1 = \lambda_2 = -2, v = (1, 0, 3)$ and $\lambda_3 = 1, v_3 = (1, -1, 2)$

Total of marks: 10