Week 11: LU decomposition

1. MULTI Single

The matrix

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -5 & 0 & 1 \end{bmatrix}$$

represents an elementary row operation. What is the determinant of T?

(a) $\det(T) = -1$
(b) $\det(T) = 0$
(c) $\det(T) = -5$
(d) $\det(T) = 1$

2. MULTI Single

A matrix A admits an LU decomposition if it can be written as $A = LU$, with L lower triangular, U upper triangular. A matrix A admits an LUP decomposition if it can be written as $PA = LU$, with L lower triangular, U upper triangular, and P a matrix that reorders rows. Which of the following is true?

(a) Any Hermitian matrix admits an LU decomposition.
(b) Any square matrix admits an LUP decomposition.
(c) A matrix admits an LU decomposition if and only if it is invertible.
(d) Any square matrix admits an LU decomposition.

3. MULTI Single

Consider the Hermitian matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

Is

$$A = LL^*$$

with

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

a valid Cholesky decomposition?

(a) No.
(b) Yes.

4. MULTI Single

A matrix A admits an LU decomposition if it can be written as $A = LU$, with L lower triangular, U upper triangular. A matrix A admits an LUP decomposition if it can be written as $PA = LU$, with L lower triangular, U upper triangular, and P a matrix that reorders rows. Which of the following is true for the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}?$$

(a) A admits an LUP decomposition, but not an LU decomposition.
(b) A admits an LU decomposition, but not an LUP decomposition.
(c) A admits neither an LUP decomposition, nor an LU decomposition.
(d) A admits both an LUP decomposition, and an LU decomposition.

5. MULTI Single

Does the matrix

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 6 & 3 & 2 \\ 0 & 3 & 8 & 5 \\ 0 & 2 & 5 & 2 \end{bmatrix}?$$

have a Cholesky decomposition?

(a) No.
(b) Yes.

6. MULTI Single

Consider the matrices

$$S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Check whether $S = LU$ is a valid LU decomposition. If the decomposition is valid, then use L and U to compute $\det(S)$.

(a) $S = LU$ and $\det(S) = 2$
(b) $S = LU$ and $\det(S) = 1$
(c) $S \neq LU$
(d) $S = LU$ and $\det(S) = -1$

7. MULTI Single

Compute the LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

such that all diagonal entries of L are one. What are the diagonal entries of U? What is the entry below the diagonal in L?

(a) LU decomposition is not possible.
(b) U has diagonal entries $-1, 2$ and the entry below the diagonal in L is 1.
(c) U has diagonal entries $1, -2$ and the entry below the diagonal in L is 3.
(d) U has diagonal entries $-1, -2$ and the entry below the diagonal in L is 2.

8. MULTI Single

Compute the LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

such that the diagonal entries of L are 3 and 5. What are the diagonal entries of U? What is the entry below the diagonal in L?

(a) LU decomposition is not possible.
(b) U has diagonal entries $\frac{1}{5}, \frac{2}{3}$ and the entry below the diagonal in L is 9.
(c) U has diagonal entries $\frac{1}{5}, -\frac{2}{3}$ and the entry below the diagonal in L is -9.
(d) U has diagonal entries $\frac{1}{3}, -\frac{2}{5}$ and the entry below the diagonal in L is 9.

9. MULTI Single

Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & -3 & 1 \\ 0 & 1 & 2 & 2 \\ -4 & 0 & 9 & 2 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

Perform an LU decomposition on A to obtain lower and upper triangular matrices L and U such that $A = LU$ and L has ones on the diagonal. What are the diagonal elements of U? What are the entries in the diagonal below the main diagonal of L?

(a) The diagonal of U has $2, 1, 3, 3$ and the entries below the diagonal of L are $0, 0, 0$.
(b) The diagonal of U has $-2, -1, -3, -3$ and the entries below the diagonal of L are $-2, -1, 0$.
(c) The diagonal of U has $2, 1, 3, -3$ and the entries below the diagonal of L are $0, 0, 1$.
(d) LU decomposition is not possible.

10. MULTI Single

Consider the system of equations

$$\begin{aligned} 6x + 18y + 3z &= 3 \\ 2x + 12y + z &= 19 \\ 4x + 15y + 3z &= 0. \end{aligned}$$

Find an LU decomposition of the associated matrix and then solve the system. What is the determinant of the associated matrix? What is the value of xyz?

(a) $\det = 18$, $xyz = 99$
(b) $\det = 36$, $xyz = -33$
(c) $\det = 36$, $xyz = 99$
(d) $\det = -18$, $xyz = 33$

Total of marks: 10