Week 2: Sequences and Series

1. [MULTI] [Single]

   Let $a_n = \sqrt{n+1} - \sqrt{n}$. Which of the following is true?

   (a) $\lim\limits_{n\to\infty} a_n = -1$

   (b) $\lim\limits_{n\to\infty} a_n = 1$

   (c) The sequence a_n diverges.

   (d) $\lim\limits_{n\to\infty} a_n = 0$

2. [MULTI] [Single]

   Let $a_n = \dfrac{n^2}{n+1}$. Which of the following is true?

   (a) $\lim\limits_{n\to\infty} a_n = 1$

   (b) The sequence a_n diverges.

   (c) $\lim\limits_{n\to\infty} a_n = -1$

   (d) $\lim\limits_{n\to\infty} a_n = 0$

3. [MULTI] [Single]

   Consider the sequence $a_n = \sqrt{n}$. Which of the following is true?

   (a) $\lim\limits_{n\to\infty} a_n = 1$

   (b) The sequence a_n diverges.

   (c) $\lim\limits_{n\to\infty} a_n = 0$

   (d) a_n is a Cauchy sequence.

4. [MULTI] [Single]

   Consider the sequence $a_n = (-1)^n - \dfrac{1}{n}$. Which of the following is true?

   (a) $\sup\{a_n : n \in \mathbb{N}\} = -1$

   (b) $\sup\{a_n : n \in \mathbb{N}\} = \dfrac{3}{2}$

   (c) $\sup\{a_n : n \in \mathbb{N}\} = 1$

   (d) $\sup\{a_n : n \in \mathbb{N}\} = 0$

5. [MULTI] [Single]

   Determine whether the series $1 + 0.4 + 0.16 + 0.064 + \ldots$ converges or diverges. If it is convergent, what is the sum?

   (a) The series converges to 1.63.

   (b) The series converges to $\dfrac{5}{3}$.

   (c) The series converges to $\dfrac{5}{2}$.

   (d) The series diverges.

6. MULTI Single

   Determine whether the series $3 + 2 + \dfrac{4}{3} + \dfrac{8}{9} + \dots$ converges or diverges. If it is convergent, what is the sum?

   (a) The series diverges.
   (b) The series converges to 7.22.
   (c) The series converges to 9.
   (d) The series converges to $\dfrac{15}{2}$.

7. MULTI Single

   Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \dfrac{10^k}{(-9)^{k-1}}$ converges or diverges. If it is convergent, what is the sum?

   (a) The series converges to $-\dfrac{100}{9}$.
   (b) The series converges to 11.23.
   (c) The series converges to -90.
   (d) The series diverges.

8. MULTI Single

   What is the radius of convergence ρ for the power series $\displaystyle\sum_{k=1}^{\infty} \dfrac{(-1)^{k-1} x^k}{k^3}$?

   (a) $\rho = 0$
   (b) $\rho = \dfrac{1}{2}$
   (c) $\rho = 1$
   (d) $\rho = \infty$

9. MULTI Single

   Let $x \in \mathbb{R}$ and consider the series $\displaystyle\sum_{k=1}^{\infty} k!(2x - 1)^k$. For what value of x does the series converge?

   (a) The series converges for $x = 0$.
   (b) The series converges for all $x \in \mathbb{R}$.
   (c) The series converges for $x = \dfrac{1}{2}$.
   (d) The series does not converges for any $x \in \mathbb{R}$.

10. MULTI Single

    What is the radius of convergence ρ for the power series $\displaystyle\sum_{k=1}^{\infty} \dfrac{2^k x^k}{k + 1}$?

    (a) $\rho = \dfrac{1}{2}$
    (b) $\rho = \infty$

      (c) $\rho = 0$

      (d) $\rho = 1$

Total of marks: 10