Week 3: Limits of Functions and Continuity

1. MULTI Single

   Which of the following is a horizontal asymptote of the function

   $$y = \frac{4x}{\log(|x|^7) + 7x}?$$

   (a) $y = 0$
   (b) $y = \frac{4}{7^2}$
   (c) The function has no horizontal asymptote
   (d) $y = \frac{4}{7}$

2. MULTI Single

   Find all vertical asymptotes of $y(x) = \sin\left(\frac{1}{x}\right) \cdot x^2 + \frac{1}{x-2}$.

   (a) $\{x = 2\}$
   (b) $\{\}$
   (c) $\{x = 0, x = 2\}$
   (d) $\{x = 0\}$

3. MULTI Single

   Find all horizontal asymptotes of $y(x) = \frac{\left(\ln\left(\frac{1}{x}\right) + \ln(x)\right) \cdot (x^2 + x + 2x) + x + \ln(x)}{x}$

   (a) $\{y = 1\}$
   (b) $\{\}$
   (c) $\{y = 0\}$
   (d) $\{y = 2\}$

4. MULTI Single

   Evaluate the limit:

   $$\lim_{v \to 2} \frac{2 - v}{\frac{1}{2} - \frac{1}{v}}$$

   (a) 2
   (b) -1
   (c) 4
   (d) -4

5. MULTI Single

   Evaluate the limit:

   $$\lim_{y \to 0} \frac{\sqrt{2 + y} - \sqrt{2 - y}}{4y}$$

   (a) $\sqrt{2}$
   (b) $\frac{1}{4\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$
(d) 1

6. MULTI Single

Evaluate the limit:

$$\lim_{x \to 0} \left(\sqrt{x} \ln x + e^x x^3 \right).$$

(a) $-\infty$
(b) $+\infty$
(c) 1
(d) 0

7. MULTI Single

Let

$$f(x) := \begin{cases} kx + 7 & \text{for } x \geq 2, \\ x^2 + 19 & \text{for } x < 2. \end{cases}$$

For what value of k is $\lim_{x \to 2} f(x)$ defined?

(a) 8
(b) 4
(c) 2
(d) 1

8. MULTI Single

Evaluate the limit:

$$\lim_{x \to 0} \frac{12^x - 1}{x}$$

(a) $1/\ln(12)$
(b) 12
(c) $\ln(12)$
(d) 0

9. MULTI Single

Evaluate the limit:

$$\lim_{N \to \infty} \sum_{k=1}^{N} \frac{1}{k^2 + k}$$

(a) 2
(b) 4/3
(c) 1
(d) 9/8

10. MULTI Single

In class we discussed the Extreme Value Theorem: If $f : [a, b] \to \mathbb{R}$ is continuous, then f assumes its minimum and maximum. Which of the following is true?

(a) The theorem applies to $f : [1, 2] \to \mathbb{R}, f(x) = \frac{1}{x}$.

(b) The theorem applies to $f : \mathbb{R} \to \mathbb{R}, f(x) = \dfrac{x^2+1}{x^2+2}$.

(c) The theorem applies to $f : \mathbb{R} \to \mathbb{R}, f(x) = e^x$.

(d) The theorem applies to $f : (0,1) \to \mathbb{R}, f(x) = \dfrac{1}{x}$.

Total of marks: 10