Week 4: Derivatives

1. MULTI Single

Let $f(x)$ be a differentiable function. Now consider

$$f_1(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}, \qquad f_2(x_0) = \lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0}$$

(a) f_1 defines a derivative while f_2 does not
(b) f_2 defines a derivative while f_1 does not
(c) Neither f_1 or f_2 define the derivative of f
(d) Both f_1 and f_2 define the derivative of f

2. MULTI Single

Calculate $\frac{\mathrm{d}}{\mathrm{d}t}\left[a^t\right]$ where $a > 0$ is a constant.

(a) ta^{t-1}
(b) $a^t \ln(a)$
(c) $a^t + a$
(d) a^t

3. MULTI Single

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{e^x}{x^2}\right)$.

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{e^x}{x^2}\right) = \frac{e^x}{x^4}$
(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{e^x}{x^2}\right) = \frac{e^x(x-2)}{x^3}$
(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{e^x}{x^2}\right) = \frac{e^x}{x^2}$
(d) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{e^x}{x^2}\right) = \frac{e^x(x^2+1)}{x^3}$

4. MULTI Single

Given that $\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\sinh(x) = \frac{e^x - e^{-x}}{2}$, which of the following is true?

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = -\cosh(x)$
(b) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = -\sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \cosh(x)$
(c) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = \sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = \cosh(x)$
(d) $\frac{\mathrm{d}}{\mathrm{d}x}\cosh(x) = -\sinh(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\sinh(x) = -\cosh(x)$

5. MULTI Single

Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(a^x + a^{-x}\right)\right]$ where $a > 0$ is a constant.

(a) $\dfrac{a^x - a^{-x}}{a^x + a^{-x}}$
(b) $\dfrac{a^x - a^{-x}}{a^x + a^{-x}} \ln a$
(c) $\dfrac{a^x + a^{-x}}{a^x - a^{-x}} \ln a$
(d) $\dfrac{a^x + a^{-x}}{a^x - a^{-x}}$

6. MULTI Single

Calculate $\dfrac{\mathrm{d}^3}{\mathrm{d}x^3}\left[x^4 e^x\right]$, i.e., the third derivative of the function.

(a) $e^x(x^4 + 12x^3 + 36x^2 + 40x)$
(b) $e^x(x^4 + 12x^3 + 24x^2 + 40x)$
(c) $e^x(x^4 + 12x^3 + 36x^2 + 24x)$
(d) $e^x(x^4 + 12x^3 + 24x^2 + 24x)$

7. MULTI Single

The Softplus function is defined as

$$\text{Softplus}(x) = \ln(1 + e^x).$$

What is the derivative of Softplus and where is it defined?

(a) Softplus is not differentiable
(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\text{Softplus}(x) = \left\{\begin{array}{ll} 0, & \text{if } x \leq 0 \\ 1, & \text{else} \end{array}\right\}$ and it is defined on $\mathbb{R}\backslash\{0\}$
(c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\text{Softplus}(x) = \dfrac{e^x}{x}$ and is defined on $\mathbb{R}\backslash\{0\}$
(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\text{Softplus}(x) = \dfrac{e^x}{1 + e^x}$ and is defined on $\mathbb{R}$

8. MULTI Single

Choose the expression equivalent to $\displaystyle\sum_{n=0}^{\infty} n \cdot x^n$.

Hint: Recall that $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ and use differentiation.

(a) $\left(\dfrac{1}{1-x}\right)^2$
(b) $-\dfrac{x}{1-x^2}$
(c) $\dfrac{x}{(1-x)^2}$
(d) $\dfrac{x}{1-x}$

9. MULTI Single

Find the derivative of $e^{3x} \cos 4x$.

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big(e^{3x}\cos 4x\Big) = e^{3x}(4\cos 4x - 3\sin 4x)$.
(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big(e^{3x}\cos 4x\Big) = e^{3x}(3\cos 4x + 4\cos 3x)$.
(c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big(e^{3x}\cos 4x\Big) = e^{3x}(3\cos 4x - 4\sin 4x)$.
(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big(e^{3x}\cos 4x\Big) = e^{3x}(3\cos 4x + 4\sin 4x)$.

10. MULTI Single

Consider the equation $\sin(y) + y^3 = 6 - x^3$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ by implicit differentiation.

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-3x^2}{\cos(y) + 3y^2}$
(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (-3x^2)(\cos(y) + 3y^2)$
(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-3x^2}{\cos(y) + y}$
(d) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-3x^2}{\sin(y) + 3y^2}$

Total of marks: 10